Parallel Ferroresonance Circuit Analysis by Chua-type Magnetization Model

Yoshifuru Saito Member (Hosei University, ysaito@hosei.ac.jp)
Iliana Marinova Non-member (Technical University of Sofia, iliana@tu-sofia.bg)
Hisashi Endo Member (Hitachi Ltd, hisashi.endo.fa@hitachi.com)

Keywords : Chua-type magnetization model, parallel ferroresonant circuit, chaotic phenomena, characteristic values

Various types of electrical apparatus using magnetic materials have been developed. Due to nonlinear properties of magnetic material, e.g., magnetic saturation, hysteresis, eddy current, etc.; the electrical apparatus occasionally exhibits complex responses that can not be anticipated and calculated easily. In the design of modern magnetic devices, prediction of various responses to the complex input signals is of paramount importance to prevent the troubles of devices. Nevertheless, any of the deterministic methodologies has not been yet proposed to do this manly caused by the complex magnetization behaviors of inductors.

To clarify the regularity of chaotic behaviors in the up-to-date power magnetic devices, this paper carries out transient analysis in a parallel ferroresonant circuit exactly taking into account the magnetic hysteresis, saturation, aftereffects, and frequency dependence of ferromagnetic material properties. To extract the system regularity, the characteristic values of the state variable equations for the ferroresonant circuit are calculated in each step in the calculation period. It is revealed that the changes of characteristic values have no hysteretic properties, even though chaotic phenomenon is exhibiting.

To carry out transient analysis of ferroresonant circuit, we employ a Chua-type magnetization model representing dynamic constitutive relation between magnetic field H (A/m) and flux density B (T) as

$$H = \frac{1}{\mu}B + \frac{1}{s}\left(\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right) \qquad (1)$$

where μ, μ_r, and s denote permeability (H/m), reversible permeability (H/m), and hysteresis parameter (Ω/m), respectively.

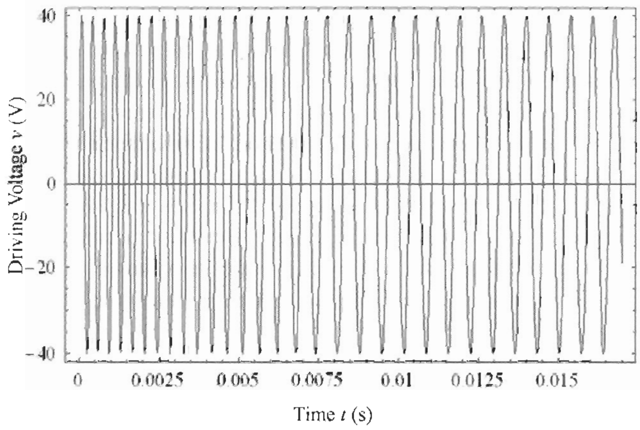
A significant feature of these parameters is that they are determined independently to the past magnetization histories because of the ideal magnetization curve approach, as pointed out by Bozorth. The permeability μ and the reversible permeability μ_r are obtained from the ideal magnetization curve and minor loops respectively when measuring the ideal magnetization curves.

When B=0, s is determined by.

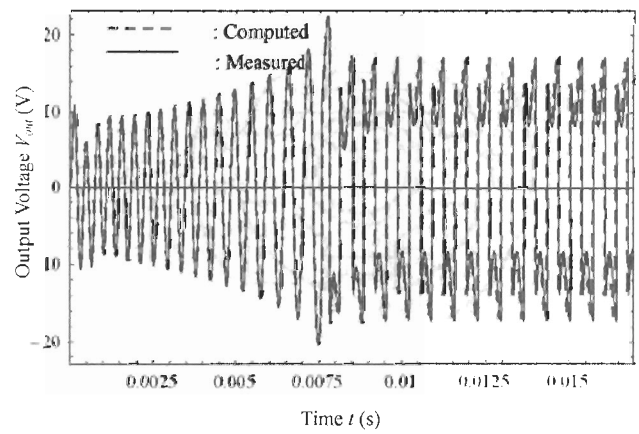
$$s = \frac{1}{H_c}\left(\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right) \qquad (2)$$

where μ_r takes maximum value and the applied field H corresponds to coercive force H_c. Thus, determination of parameter s in Eq. (2) requires the measurements of the dB/dt and dH/dt. The validity of this model has been verified by the precise experiments of magnetization characteristics excepting for anisotropic materials and permanent magnets of typical materials.

Figure 1 illustrates dV_{out}/dt versus V_{out}, exhibiting chaos-like behavior not tracing the same locus while the frequency of the driving voltage v is fixed at t =7.8 ms. Let us compare the series and parallel ferroresonant phenomena. At the beginning of resonance, either output response drastically increases. If the driving voltage is fixed when the ferroresonant mode is reached, we have nonlinear oscillation continuously. On Poincare diagrams, the parallel ferroresonance shakes dV_{out}/dt although the frequency of driving voltage is fixed. Further, the series ferroresonance has the same nature of small shaking in the current applied to the inductor. Since dV_{out}/dt in parallel ferroresonant circuit is associated with current, then these phenomena suggest that the chaos-like flicking is closely related to a condition of input term.



(a) Driving voltage v



(b) Measured and computed output voltage V_{out}

Fig. 1. Transient analysis of the ferroresonant circuit

Paper

Parallel Ferroresonance Circuit Analysis by Chua-type Magnetization Model

Yoshifuru Saito* Member
Iliana Marinova** Non-member
Hisashi Endo*** Member

This paper studies the nonlinear response of a parallel ferroresonant circuit. To carry out a transient analysis in parallel ferroresonant circuit, we apply Chua-type magnetization model to an inductance exhibiting saturation and hysteretic nonlinear properties of ferromagnetic materials, deriving a state variable equation and solutions by the backward Euler method with automatic modification. The characteristic values of the state transition matrix are calculated in each calculation step of Euler method in order to extract the chaotic characteristics. As a result, it is clarified that the chaotic behavior in the ferroresonant circuit is greatly concerned with the magnetic aftereffect of ferromagnetic materials.

Keywords : Chua-type magnetization model, parallel ferroresonant circuit, chaotic phenomena, characteristic values

1. Introduction

Various types of electrical apparatus using magnetic materials have been developed. Due to nonlinear properties of magnetic material, e.g., magnetic saturation, hysteresis, eddy current, etc.; the electrical apparatus occasionally exhibits complex responses that can not be anticipated and calculated easily. In the design of modern magnetic devices, prediction of various responses to the complex input signals is of paramount importance to prevent the troubles of devices. Nevertheless, any of the deterministic methodologies has not been yet proposed to do this mainly caused by the complicated magnetization behaviors of inductors.

To clarify the regularity of chaotic behaviors in the up-to-date power magnetic devices, this paper carries out transient analysis in a parallel ferroresonant circuit exactly taking into account the magnetic hysteresis, saturation, aftereffects, and frequency dependence of ferromagnetic material properties[(1)-(5)]. To extract the system regularity, the characteristic values of the state variable equations for the ferroresonant circuit are calculated in each step in the calculation period. It is revealed that the changes of characteristic values have no hysteretic properties, even though chaotic phenomenon is exhibiting.

2. Parallel Ferroresonant Circuit

2.1 Chua-type Magnetization Model To carry out transient analysis of ferroresonant circuit, we employ a Chua-type magnetization model representing dynamic constitutive relation between magnetic field H (A/m) and flux density B (T) as

$$H = \frac{1}{\mu}B + \frac{1}{s}\left(\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right) \qquad (1)$$

where μ, μ_r, and s denote permeability (H/m), reversible permeability (H/m), and hysteresis parameter (Ω/m), respectively[(3)(5)]. Figure 1 shows the measured curves giving these parameters for ferrite (TDK H5A). A significant feature of these parameters is that they are determined independently to the past magnetization histories because of the ideal magnetization curve approach, as pointed out by Bozorth[(6)]. The permeability μ and the reversible permeability μ_r are obtained from the ideal magnetization curve and minor loops respectively when measuring the ideal magnetization curves. When B=0, s is determined by

$$s = \frac{1}{H_c}\left(\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right) \qquad (2)$$

where μ_r takes maximum value and the applied field H corresponds to coercive force H_c. Thus, determination of parameter s in Eq. (2) requires the measurements of the dB/dt and dH/dt. The validity of this model has been verified by the precise experiments of magnetization characteristics excepting for anisotropic materials and permanent magnets of typical materials[(7)-(10)].

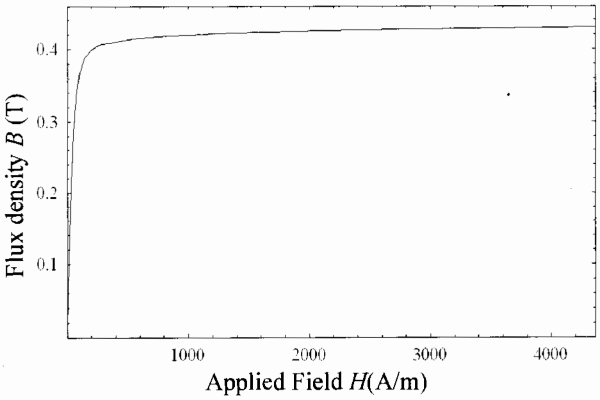
2.2 Formulation Consider a parallel ferroresonant circuit shown in Fig. 2. At first, the line integral of Eq. (1) along with flux path l yields magnetomotive force. Thus, the relation between the current i_1 and linkage flux λ of the inductor is given by

$$Ni_1 + \frac{\mu_r}{s}N\frac{di_1}{dt} = \frac{l}{\mu AN}\lambda + \frac{l}{sAN}\frac{d\lambda}{dt} \qquad (3)$$

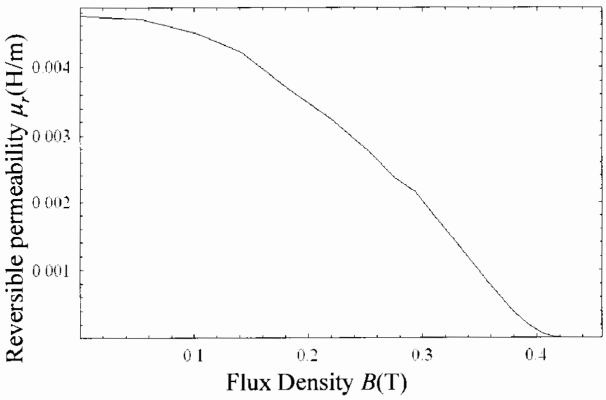
Moreover, a relation between the driving voltage source v and current i_1 is derived from the consideration of circuit connection and electromotive force $d\lambda/dt$

* Graduate School of Hosei University
3-7-2, Kajimo-cho, Koganei 184-8584
** Department of Electrical Apparatus, Technical University of Sofia
Kliment Ohridski 8, Sofia 1756 Bulgaria
*** Institute of Fluid Science, Tohoku University / Currently, Power & Industrial Systems R&D Laboratory, Hitachi Ltd.
7-2-1, Ohmika, Hitachi 319-1221

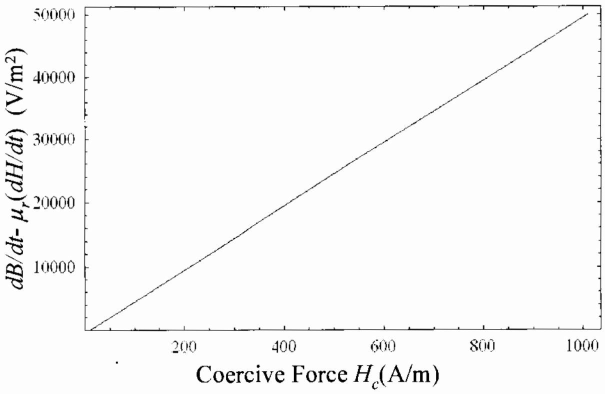
© 2008 The Institute of Electrical Engineers of Japan.



(a) Magnetization curve giving permeability μ



(b) Reversible permeability μ_r



(c) The curve giving hysteresis parameter s

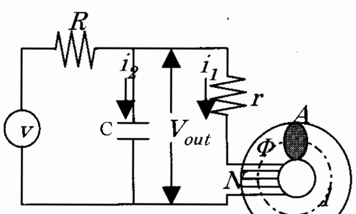
Fig. 1. Parameters of Chua-type magnetization model (Measured: TDK H5A)

$$i_1 = \frac{1}{r}\left(V_{out} - \frac{d\lambda}{dt}\right) \quad \cdots\cdots (4)$$

Second, substituting Eq. (3) into Eq. (4) yields the state equations

$$\frac{\mu_r N}{sr}\frac{d^2\lambda}{dt^2} = \left\{-\frac{N}{r} + \frac{\mu_r N}{sR^2C} - \frac{l}{sAN}\right\}\frac{d\lambda}{dt} - \frac{l}{\mu AN}\lambda + \left\{\frac{N}{r} - \frac{\mu_r N}{sr} - \left(\frac{1}{RC} + \frac{1}{rC}\right)\right\}V_{out} + \frac{\mu_r N}{srRC}v \quad \cdots\cdots (5)$$

$$\frac{dV_{out}}{dt} = \frac{1}{rc}\frac{d\lambda}{dt} - \left(\frac{1}{RC} + \frac{1}{rC}\right)V_{out} + \frac{1}{RC}v \quad \cdots\cdots (6)$$



A : cross-sectional area (m^2)
V_{out} : output voltage (V)
l : flux path length (m)
N : number of coil turns
R : resistance (Ω)
r : internal resistance (Ω)
v : driving voltage (V)
Φ : magnetic flux (Wb)
λ : linkage flux (Wb) (=$N\Phi$)

Fig. 2. Parallel ferroresonant circuit

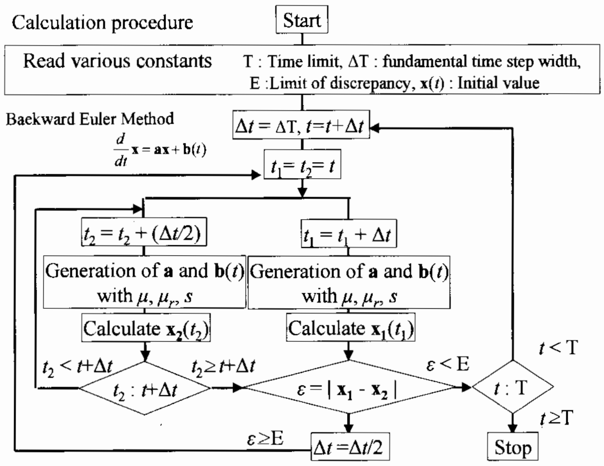


Fig. 3. Flowchart of the calculation with the adaptive step size control

Finally, the state Eqs. (5) and (6) yield a systm of state variable equations having 3 × 3 square state transition matrix **a**

$$\frac{d}{dt}\begin{pmatrix}\lambda \\ \frac{d\lambda}{dt} \\ V_{out}\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33}\end{pmatrix}\begin{pmatrix}\lambda \\ \frac{d\lambda}{dt} \\ V_{out}\end{pmatrix} + \begin{pmatrix}0 \\ u_2 \\ u_3\end{pmatrix} \quad \cdots\cdots (7)$$

or

$$\frac{d}{dt}\mathbf{x} = \mathbf{ax} + \mathbf{b}(t) \quad \cdots\cdots (8)$$

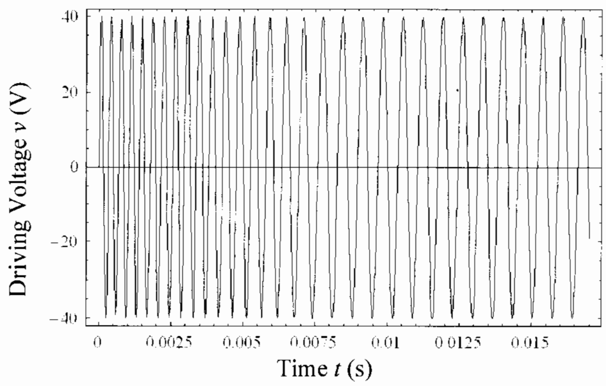
where the elements a_{21}, a_{22}, … , u_2 and u_3 in Eq. (7) are determined by Eqs. (5) and (6).

2.3 Backward Euler Method The backward Euler strategy to numerical solution of Eq. (7) makes it possible to carry out a transient analysis of ferroresonant circuit. As shown in Fig. 3, the calculation compares two solutions in each calculation step. The first one is one step solution with time step-width Δt (s), and another is two steps solution with $\Delta t/2$. Evaluating the difference between them reveals a relevant step-with for each of the caluculations. Namely, if the difference is greater than a criterion listed in Table 1, then the same period is recalculated with the modified time step width Δt=$\Delta t/2$. The iteration with this modification is carried out untill the criterion is satisfied.

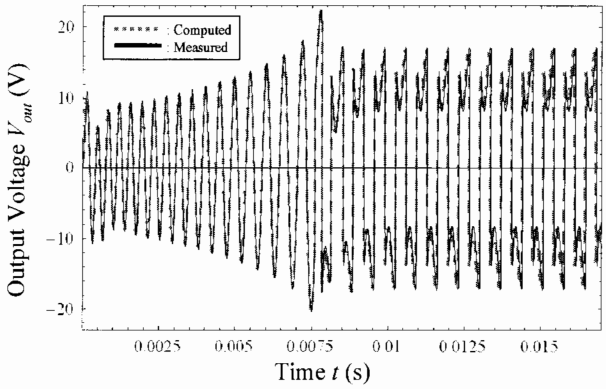
In the iterative calculation, the nonlinear parameters, shown in Fig. 1, μ and μ_r are treated as functions of flux density B and s as a function of dB/dt (7).

3. Results and Discussion

3.1 Ferroresonant Phenomenon Figure 4 shows the



(a) Frequency of the driving voltage v is decreased from 3.0 to 1.441 kHz until time t=7.8 ms



(b) Measured and computed output voltage V_{out}

Fig. 4. Transient analysis of the ferroresonant circuit

Table 1. Parameters for calculation of ferroresonant circuit

μ : permeability (H/m)	Fig. 1(a)
μ_r : reversible permeability (H/m)	Fig. 1(b)
s : hysteresis parameter (Ω/m)	Fig. 1(c)
A : cross-sectional area (m2)	48.0×10^{-6}
C : capacitance (F)	1.0×10^{-6}
l : flux path length (m)	75.4×10^{-3}
N : number of coil turns	100
R : resistance (Ω)	272.0
r : internal resistance (Ω)	0.4
ε : limit of discrepancy	1.0×10^{-5}

calculated result of Eq. (7) employing the parameters in listed Table 1. As demonstrated in Fig. 4 (b), the experimentally obtained output voltage well agrees with calculated one. As shown in Fig. 5, the frequency of the driving voltage v in Fig. 4 (a) is decreased from 3.0 to 1.441 kHz until time t =7.8 ms in order to observe its ferroresonant process. Around this moment, the output voltage V_{out} drastically increases, exhibiting the typical ferroresonant phenomena.

3.2 Chaotic Behavior Figure 6 illustrates dV_{out}/dt versus V_{out} obtained from Fig. 4 (b), exhibiting chaos-like behavior not tracing the same locus while the frequency of the driving voltage v is fixed at t =7.8 ms. Let us compare the series and parallel ferroresonant phenomena(11). At the beginning of resonance, either output response drastically increases. If the driving voltage is fixed when the ferroresonant mode is reached, we have nonlinear oscillation continuously. On Poincare diagrams,

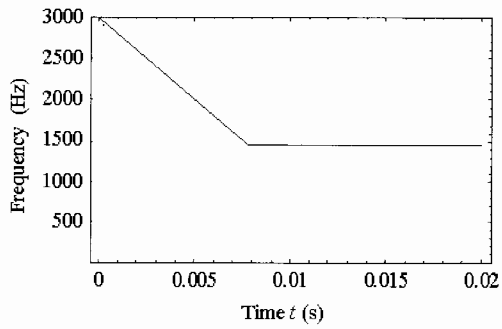


Fig. 5. Frequency variation of source voltage v

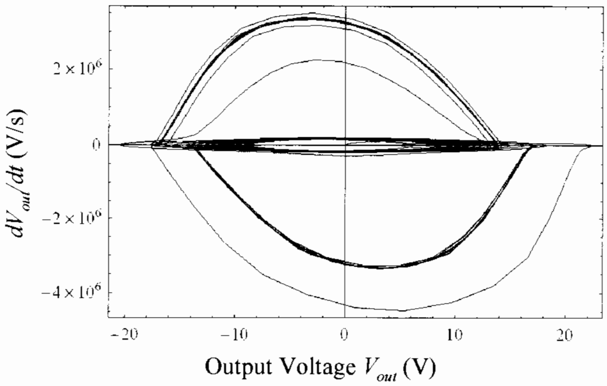


Fig. 6. Poincare diagram during ferroresonant mode

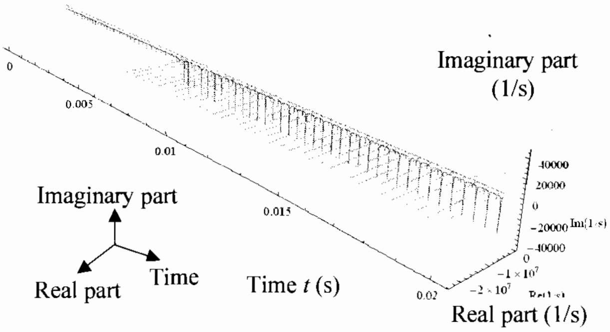
the parallel ferroresonance shakes dV_{out}/dt although the frequency of driving voltage is fixed. Further, the series ferroresonance reported in Ref. (11) has the same nature of small shaking in the current applied to the inductor. Since dV_{out}/dt in parallel ferroresonant circuit is associated with current, then these phenomena suggest that the chaos-like flicking is closely related to a condition of input term of Eq. (7) or Eq. (8).

3.3 System Regularity To consider the state of ferroresonant system in detail, we calculate the characteristic values of the state transition matrix **a** in Eq. (7) in each of the calculation steps. The characteristic value analysis could be applied to only the linear system. So that we essentially assume this nonlinear system to be a piecewise linear system in each of the calculation steps for solving Eq. (7).

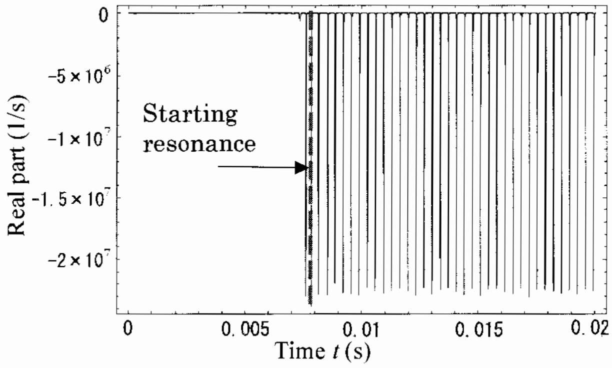
Figure 6 shows the loci of characteristic values derived from the state transition matrix **a** in Eq. (7) assuming a to be linear in each calculation step. Since **a** is 3×3 square matrix, we have three characteristic values. Fig.7 shows time versus characteristic values, presenting that these are tracing on the regular loci. Meanwhile, the output voltage locus exhibits a chaotic flicking. One of the causes of this chaotic flicking is an equivalent coercive force H_c=$(\mu_r/s)dH/dt$ in the Chua type magnetization model (1). This instanteneously leads to the positive real part of the characteristic values as shown in Fig.7 (b).

Thus, this is the cause of the instable chaos-like flicking. The characteristic values at the other timing are all on the left half plane on a complex coordinate system.

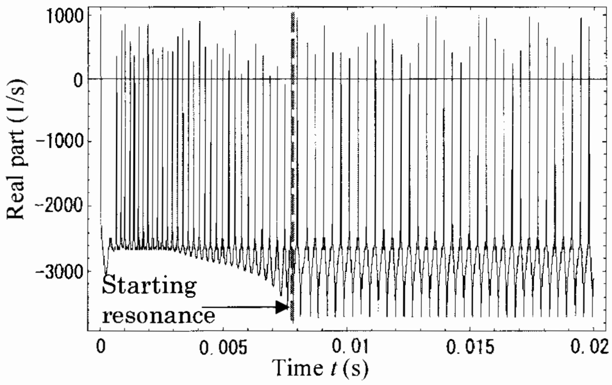
Let us consider the characteristic values by decomposed into the real and imaginary parts. Figs. 7 (b)-(d) express the characteristic values given as pure real number, complex number in real part, and complex number in imaginary part, respectively. Since the complex number characteristic values are always given as



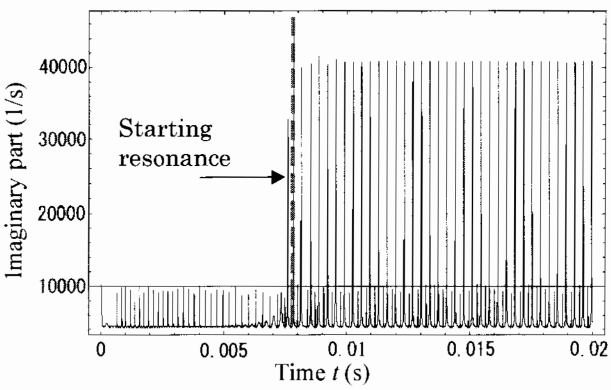
(a) Time varying of the characteristic values



(b) Pure real number part



(c) Real part of complex number



(d) Imaginary part of complex number

Fig. 7. Characteristic values derived from the state transition matrix

complex conjugate pairs, then we omit one of them. The real parts in Figs. 7 (b) and (c) regularly oscillate with changes of amplitude after the resonance starts, affecting nonlinear output response in V_{out} when resonance occurring. The imaginary part in Fig. 7 (d) oscillates in nearly constant amplitude and drastically increases after the rouse of resonance. This gives that the state of resonance depends on the imaginary parts. Observing the characteristic values makes it possible to predict the ferroresonant phenomena.

4. Conclutions

In this paper, we have derived the state variable equations of the parallel ferroresonance circuit employing the Chua-type magnetization model, and carried out the transient analysis of the parallel ferroresonant circuit. The characteristic value analysis of the state transition matrices obtained in every calculation step of Euler method has elucidated that the cause of chaotic flicking is an equivalent coercive force $H_{c'}=(\mu_r/s)dH/dt$.

As described above, it is revealed that our approach employing the Chua-type magnetization model clarifies the precise processes of the parallel ferroresonance phenomenon.

Acknowledgments

The authors are cknowledged to Mr. Y. Tanaka, Hosei University (currently Nihondensan Co. LTD) for his effort to obain the experimental as well as calculated results on this paper.

(Manuscript received Nov. 22, 2007, revised Jan. 31, 2008)

References

(1) F. Liorzou, B. Phelps, and D. L. Atherton : "Macroscopic models of magnetization", *IEEE Trans. Magnetics.*, Vol.36, pp.418-427 (2000-3)

(2) E. Della Torre : Magnetic Hysteresis, IEEE Press Piscataway, NJ (1999)

(3) A. Ivanyi : Hysteresis Models in Electromagnetic Computation. Akademiai Kiado, Budapest, Hungary (1997)

(4) L. O. Chua and K. A. Stromsmoe : "Lumped circuit models for nonlinear inductor exhibiting hysteresis loops", *IEEE Trans. Circuit Theory*, Vol.CT-17, pp.564-574 (1970-4)

(5) Y. Saito, M. Namiki, and S. Hayano : "A magnetization model for computational magnetodynamics", *J. Appl. Phys.*, Vol.69, No.8, pp.4614-4616 (1991)

(6) Y. Saito, M. Namiki, and S. Hayano : "A representation of magnetization characteristics and its application to the ferroresonance circuits", *J. Appl. Phys.*, Vol.67, No.9, pp.4738-4740 (1990)

(7) R. M. Bozorth : Ferromagnetism, Princeton NJ (1951)

(8) Y. Saito, S. Hayano, and N. Tsuya : "Experimental verification of a Chua type magnetization model", *IEEE Trans. Magnetics*, Vol.MAG-25, pp.2968-2970 (1989-7)

(9) Y. Saito, K. Fukushima, S. Hayano, and N. Tsuya : "Application of a Chua-type model to the loss and skin effect calculations", *IEEE Trans. Magnetics*, Vol.MAG-23, pp.3569-3571 (1987-9)

(10) Y. Saito, S. Hayano, Y. Kishino, K. Fukushima, H. Nakamura, and N. Tsuya : "A representation of magnetic aftereffect", *IEEE Trans. Magnetics*, Vol.MAG-22, pp.647-649 (1986-9)

(11) H. Endo, I. Marinova, T. Takagi, S. Hayano, and Y. Saito : "Dynamics on ferroresonant circuit exhibiting chaotic phenomenon", *IEEE Trans. Magnetics*, Vol.40, No.2, pp.868-871 (2004-3)

Yoshifuru Saito

(Member) was born in Fukuoka, Japan on July 24, 1946. Professor Saito attended Hosei University (B.E. 1969, M.E. 1971, Ph.D. 1975). Dr. Saito was an assistant research fellow (1975-76), lecturer (1976-78) and was appointed Associate Professor (1978-87) and Professor (1987-) in the Electrical Engineering Department at Hosei University. Currently, he is a Professor of the graduate school of system designing.

Iliana Marinova

(Non-member) was born in Pleven, Bulgaria on June 10, 1959. She received a Ph.D. degree in electrical engineering from Technical University of Sofia, Bulgaria in 1989, and is presently an associate professor at Technical University of Sofia. She has worked on inverse problems in electromagnetism and biomagnetism, modelling and visualization of electromagnetic fields, optimal design and investigation of electromagnetic devices. IEEE Magnetics society, International Compumag society member.

Hisashi Endo

(Member) was born in Kanagawa on July 7, 1976. He received his BE, ME, and Ph.D. from Hosei University in 1999, 2001, and 2004, respectively. He worked at Institute of Fluid Science, Tohoku University from 2002 to 2006. Currently, he is working at Power & Industrial Systems Laboratory, Hitachi Ltd. from 2007. His research interests include nondestructive evaluation, image analysis and electromagnetic computation.