Applications of Electromagnetic Phenomena in Electrical and Mechanical Systems
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Generalized vector sampled pattern matching method for inverse parameter problems

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ABSTRACT: We apply the generalized vector sampled pattern matching (GVSPM) method to an inverse parameter problem. For the inverse parameter problems, it is possible to obtain a unique solution when measuring the fields ideally, such as the computed tomography (CT). However, most of the inverse problems are reduced into solving for an ill-posed system of equations whose solution is not uniquely determined. The GVSPM introduced in this paper enable us to select the physically existing solution among possible ones.

1 INTRODUCTION

In 1917, Radon gave the mathematical background of computed tomography (CT). If the projection of the object that exists in a plane is obtained, the form of the object can be reconstructed. This is the fundamental idea of tomography, and it was applied to X-ray CT or Magnetic Resonance Imaging (MRI). More similar methods of tomography are extensively developed for the medicine field. Instead of the light projection, X-ray CT employs X-ray absorption rate when X-rays are irradiated at an object, and also, MRI employs the absorption rate of the microwave. Because in both cases absorption rate of X-ray or microwave is used directly, the mathematical basis is also clear. Therefore, it has been developed comparatively early and used for practical use [1].

Electrical impedance tomography (EIT) utilizes surface electrical potential distribution of around target when injecting electrical current to the object. Final target of EIT is to obtain conductivity distribution of the object as a tomography. In case of the X-ray or MRI tomography, X-ray and microwave go straight to the object so that information of object can be expressed in a simple mathematically relation. However, in case of EIT, the surface electrical potential distribution of the object caused by injecting current could be obtained as a solution of Laplace equation. This leads EIT to a functional tomography depending on the medium parameter as well as boundary condition. EIT device itself can be composed in the quite simple mechanical as well as electrical structures. Because of its functional nature, the EIT is essential to evaluate a solution of the Laplace equation with unknown medium parameter [2]. This means that realization of EIT necessaries a solution of an inverse parameter problem.

In this paper, we examine an inverse parameter problem for which the resistance distribution is evaluated from the nodal voltages when injecting the currents to the planar electrical circuits in order to carry out the basic development of EIT.

This paper consists of five chapters and the first chapter is a foreword. The second chapter discusses the lumped electrical circuit model of EIT. The third chapter is generalized vector sampled pattern matching method that are one of the ways to solve ill posed liner system of equations encountered in the EIT problem. The fourth chapter is a simulation that employs a lumped electrical circuit model. Also, experimental verification is demonstrated. The fifth chapter is a summary.

2 Lumped electrical circuit model of EIT

For simplicity, let us consider an electrical circuit model instead of the practical EIT. As a concrete example of electrical circuit model, we consider a simple electrical circuit shown in Fig. 1.

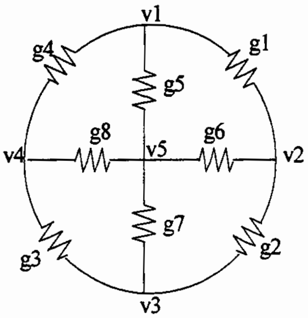

Fig.1. Electrical circuit with 5 nodes, 8 resistances.

In Fig.1, $v_1, v_2 ... v_5$ and $g_1, g_2 \cdots g_8$ are the nodal electric potentials when injecting an electric current to the circuits and electrical conductances comprising the circuit.

Injecting an electric current I to the nodes between 1 and 3 yields a following nodal equations.

$$\begin{aligned} &(v_1 - v_2)g_1 + (v_1 - v_4)g_4 + (v_1 - v_5)g_5 = I, \\ &(v_2 - v_1)g_1 + v_2 g_2 + (v_2 - v_5)g_6 = 0, \\ &- v_2 g_2 - v_4 g_3 - v_5 g_7 = -I, \\ &(v_4 - v_3)g_3 + (v_4 - v_1)g_4 + (v_4 - v_5)g_8 = 0, \\ &(v_5 - v_1)g_5 + (v_5 - v_2)g_6 + v_5 g_7 + (v_5 - v_4)g_8 = 0. \end{aligned} \tag{1}$$

(1) can be rewritten a matrix form as

$$\begin{bmatrix} v_1-v_2 & 0 & 0 & v_1-v_4 & v_1-v_5 & 0 & 0 & 0 \\ v_2-v_1 & v_2-v_3 & 0 & 0 & 0 & v_2-v_5 & 0 & 0 \\ 0 & -v_2 & -v_4 & 0 & 0 & 0 & -v_5 & 0 \\ 0 & 0 & v_4-v_3 & v_4-v_1 & 0 & 0 & 0 & v_4-v_5 \\ 0 & 0 & 0 & 0 & v_5-v_1 & v_5-v_2 & v_5 & v_5-v_4 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \\ g_7 \\ g_8 \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ -I \\ 0 \\ 0 \end{bmatrix}. \tag{2}$$

or

$\mathbf{CX} = \mathbf{Y}$.

(2) is an ill posed linear system of equation in order to evaluate the solution vector **X** with order 8, while we have 5 equations [4].

3 GENERALIZED VECTOR SAMPLED PATTERN MATCHING METHOD

3.1 Formulation

Let us consider a linear system of equations:

$$\mathbf{Y} = C\mathbf{X}, \tag{3}$$

where **Y** and **X** denote the n-th order current- and m-th order conductance- vectors, respectively. C is an n by m voltage matrix. (3) can be rewritten by

$$\begin{aligned} \mathbf{Y} &= \sum_{i=1}^{m} x_i \mathbf{C}_i, \\ \mathbf{X} &= [x_1 \quad x_2 \quad . \quad x_m]^T, \\ C &= [\mathbf{C}_1 \quad \mathbf{C}_2 \quad . \quad \mathbf{C}_m]. \end{aligned} \tag{4}$$

Normalization of (4) gives the following relationship:

$$\frac{\mathbf{Y}}{|\mathbf{Y}|} = \sum_{i=1}^{m} x_i \frac{|\mathbf{C}_i|}{|\mathbf{Y}|}\frac{\mathbf{C}_i}{|\mathbf{C}_i|} \quad \text{or} \quad \mathbf{Y}' = C'\mathbf{X}', \tag{5}$$

where the prime (') denotes the normalized quantities. (5) means that the normalized input vector **Y**' is obtained as a linear combination of the weighted solutions $x_i|\mathbf{C}_i|/|\mathbf{Y}|$, i=1, 2,..., m, with the normalized column vectors $\mathbf{C}_i/|\mathbf{C}_i|$, i= 1, 2,..., m. It should be noted that the solution **X** could be obtained when an inner product between **Y**' and C'**X**' becomes 1. This is the key idea of the GVSPM method.

3.2 Objective Function

Define a function f derived from an angle between the input vector **Y** and $C\mathbf{X}^{(k)}$ given in terms of the k-th iterative solution $\mathbf{X}^{(k)}$, as given by

$$f\left(\mathbf{X}^{(k)}\right) = \frac{\mathbf{Y}}{|\mathbf{Y}|} \bullet \frac{C\mathbf{X}^{(k)}}{\left|C\mathbf{X}^{(k)}\right|} = \mathbf{Y}' \bullet \frac{C'\mathbf{X}'^{(k)}}{\left|C'\mathbf{X}'^{(k)}\right|}. \tag{6}$$

Then the solution $\mathbf{X}^{(k)}$ is obtained when the function $f(\mathbf{X}^{(k)})$ converges to

$$f\left(\mathbf{X}^{(k)}\right) \to 1. \tag{7}$$

This is the objective function of the GVSPM solution.

3.3 Iteration Algorithm

Let $\mathbf{X}'^{(0)}$ be an initial solution vector given by

$$\mathbf{X}'^{(0)} = C'^T \mathbf{Y}', \tag{8}$$

then the first deviation vector $\Delta \mathbf{Y}'^{(1)}$ is obtained as

$$\Delta \mathbf{Y}'^{(1)} = \mathbf{Y}' - \frac{C'\mathbf{X}'^{(0)}}{\left|C'\mathbf{X}'^{(0)}\right|}, \tag{9}$$

when the deviation $\Delta \mathbf{Y}'$ becomes zero vector, the objective function (7) is automatically satisfied. Modification by the deviation vector $\Delta \mathbf{Y}'^{(k-1)}$ gives the k-th iterative solution vector $\mathbf{X}'^{(k)}$, namely,

$$\begin{aligned} \mathbf{X}'^{(k)} &= \mathbf{X}'^{(k-1)} + C'^{T} \Delta \mathbf{Y}'^{(k-1)} \\ &= C'^{T} \mathbf{Y}' + \left(I_m - \frac{C'^{T} C'}{\left|C'\mathbf{X}'^{(k-1)}\right|} \right) \mathbf{X}'^{(k-1)}, \end{aligned} \tag{10}$$

where I_m denotes a m by m unit matrix.

3.4 Convergence Condition

The convergence condition of the GVSPM iterative strategy is that the modulus of all characteristic values of state transition matrix in (10) must be less than 1. The state transition matrix S is given by

$$S = I_m - \frac{C'^{T} C'}{\left|C'\mathbf{X}'^{(k-1)}\right|} = I_m - \frac{C'^{T} C'}{\left|\mathbf{Y}'^{(k-1)}\right|} \quad . \tag{11}$$

Since the vector $\mathbf{Y}'^{(k-1)}$ is normalized, (11) can be rewritten by

$$S = I_m - C'^{T} C'. \tag{12}$$

Let λ be the characteristic value of the state transition matrix S, then the determinant of symmetrical matrix is obtained:

$$\left|\lambda I_m - S\right| = \begin{vmatrix} \lambda & \varepsilon_{12} & . & \varepsilon_{1m} \\ \varepsilon_{12} & \lambda & . & \varepsilon_{2m} \\ . & . & . & . \\ \varepsilon_{1m} & \varepsilon_{2m} & . & \lambda \end{vmatrix} = 0 \cdot \tag{13}$$

It is obvious that the modula of off-diagonal elements in (13) take less than 1 because of the normalized column vectors of matrix C', namely,

$$|\varepsilon_{ij}| < 1, \qquad i=1,2,\ldots,m, j=1,2,\ldots,m. \tag{14}$$

Suppose the modulus characteristic value $|\lambda|$ takes more than 1. Then the column vectors in (13) become linear independent because of (14). In such a case, the determinant in (13) is not zero so that the condition $|\lambda| < 1$ should be satisfied.

Thus, it is proofed that the GVSPM is always carried out on stable iteration [3].

4 Simulation and Experiment

4.1 Simulation

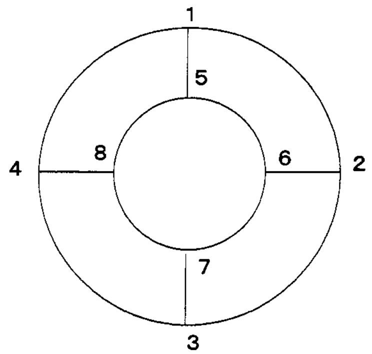

Fig.2. A simulation model.

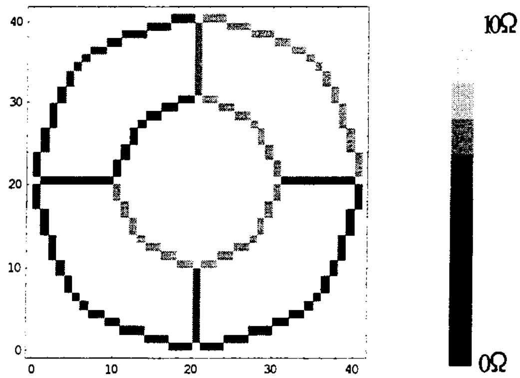

Fig.3. Assumed conductance distribution.

As a simulation example, let us consider an electric circuit model shown in Fig.2, where Fig.3 gives its conductance distribution.

When an electric current is injected to the nodes between 1 and 3 in Fig.2, application of the GVSPM to this inverse parameter problem yields the conductance distribution shown in Fig.4. In Figs. 3 and 4, the light and dark tones are corresponding to the high and small values, respectively.

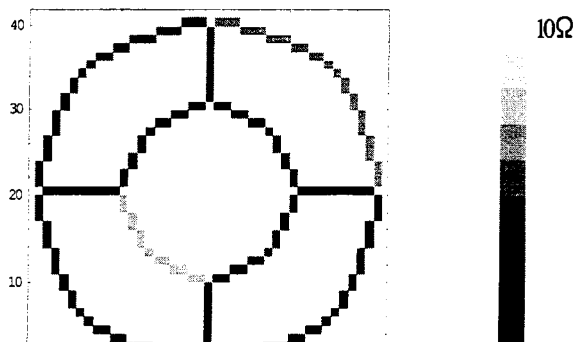

Fig.4. Conductance distribution evaluated from a system having 8×12 system matrix.

Obviously, the result in Fig.4 is not corresponding to those of Fig.3. This means that information provided by single electrical current injection is not enough to obtain the physically existing solution, even if the GVSPM is capable of obtaining a solution vector of any ill-posed system of equations.

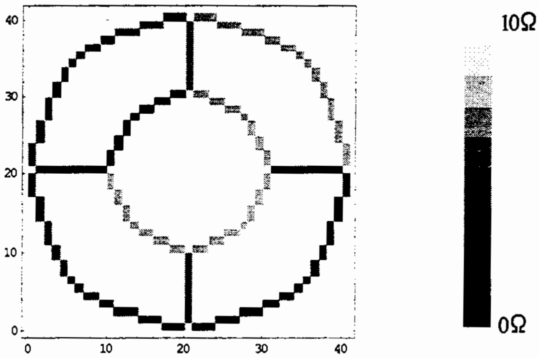

Fig.5. Conductance distribution evaluated from a system having 16×12 system matrix.

When we independently set the nodes between 1 and 3 and between 2 and 3 for current injections, we have 16 equations and 12 unknowns. Namely, we solve an ill posed liner system of equations whose system matrix is 16×12. Fig.5 shows the conductance distribution evaluated from a system having 16×12 system matrix. Comparing Figs.3 with 5, it is obvious that GVSPM has yields the physically existing conductance distribution.

Thus, it has been revealed that conductance distribution tomography is theoretically possible by increasing response information to the object.

4.2 Experimental verification

Fig.6 shows the electric circuit used for the experiment. Fig.7 shows a conductance distribution of the tested electric circuit in Fig.6.

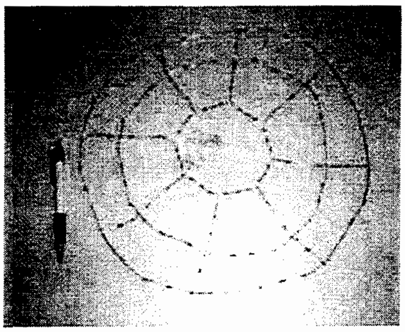

Fig.6. The circuit used for the experiment.

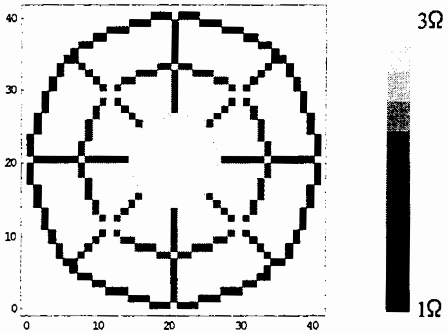

Fig.7. Conductance distribution used for the experiment.

After numbering the nodes in Fig.6 or 7 in much the same way as Fig.2, we have measured the nodal electric voltages when injecting the currents. Table 1lists the measured nodal electric voltages while the node 5 has been selected as a reference point.

In this experiment, we have evaluated the solution of a linear system of equations having 96×40 system matrix by GVSPM method to obtain conductance distribution. Fig.8 shows the evaluated conductance distribution, which does not correspond exactly compared with those of Fig.7. However, it is observed that the tendency of conductance distribution in Fig.8 is similar to those of Fig.7.

Table 1. Pair of electrodes and nodal electric voltages.

voltage[v]	electrode 5-4	electrode 5-3	electrode 5-2	electrode 5-1
v1	0.188	0.227	0.275	0.356
v2	0.225	0.281	0.355	0.261
v3	0.267	0.352	0.234	0.184
v4	0.334	0.178	0.125	0.094
v5	0.000	0.000	0.000	0.000
v6	0.070	0.077	0.083	0.098
v7	0.114	0.130	0.148	0.181
v8	0.153	0.178	0.209	0.256
v9	0.184	0.222	0.253	0.291
v10	0.217	0.261	0.289	0.247
v11	0.244	0.278	0.216	0.181
v12	0.244	0.178	0.141	0.114
v13	0.094	0.077	0.070	0.660
v14	0.094	0.097	0.100	0.114
v15	0.120	0.134	0.152	0.181
v16	0.155	0.178	0.206	0.244
v17	0.183	0.214	0.239	0.255
v18	0.209	0.242	0.250	0.230
v19	0.223	0.238	0.209	0.181
v20	0.206	0.178	0.148	0.130
v21	0.133	0.117	0.106	0.102
v22	0.116	0.117	0.120	0.130
v23	0.130	0.142	0.156	0.180
v24	0.156	0.178	0.200	0.230

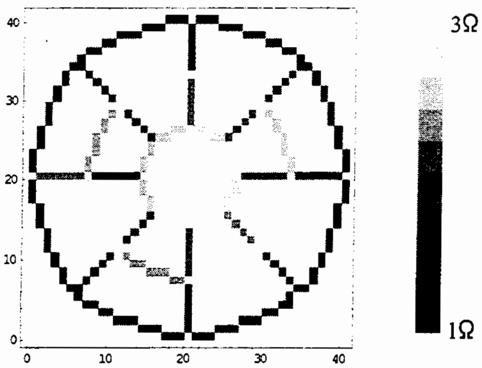

Fig.8 Conductance distribution evaluated from a system having 96×40 system matrix.

5 Conclusion

In this paper, we have examined an inverse parameter problem along with the GVSPM method as the first stage of EIT development.

As a result, we have succeeded in obtaining a reasonable result in simulation. However, practical experiment has shown that the tendency of evaluated conductance distribution is similar to those of tested one, but not possible to obtain the exact conductance distribution because of noise include in measured electric nodal voltages.

6 REFERENCES

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