Applications of Electromagnetic Phenomena in Electrical and Mechanical Systems
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Current distribution in multi-layered printed circuit board

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ABSTRACT: Principal purpose of this paper is to obtain the frequency classified current distributions on a multi-layered PCB (printed circuit board). Only inverse approach, where the current distributions could be estimated from the locally measured magnetic fields around the electronic devices, enables us to carry out this non-destructive inspection. Concretely, at first, we have measured magnetic field from the upper and bottom of a PCB. Second, we have applied Fourier transform to the output sensor signals in order to classify the measured magnetic fields. Finally, convolution of the obtained current distributions between the top and bottom sides of estimation exactly identifies the current flowing paths.

1. INTRODUCTION

Modern portable electronics are always constructed by thin shape elements such as printed circuit board (PCB) for carrying as well as handling like notebooks. Visualization of the current distributions on a printed circuit board is one of the reasonable methodologies of checking the operation and finding faults. Visualization of the currents without any decomposition falls into an inverse problem, which evaluates currents from the locally measured magnetic fields around them. However, the solution of inverse problems tends to increase the noise effects essentially accompanying the magnetic field measurements [1]. To develop a system for visualization the currents, we have tried various theoretical and experimental verifications [2,3]. In this paper, we apply our methodology to multi-layered printed circuit board operating under different frequencies. First, we measured the time domain magnetic fields at the front and the backside of PCB. Application of Fourier transform to the measured magnetic fields makes possible to classify magnetic fields into distinct time frequency components. Second, employing the generalized vector sampled pattern matching (GVSPM) method in order to solve the ill-posed inverse problems. Each of the current distributions having distinct frequency is evaluated from the top and bottom magnetic field components, respectively. Because of the multi-layered-PCB, current distribution in each of the layers should be evaluated taking the different distances from target layer to filed measured surfaces into account. Our methodology has succeeded in extracting the major components of current distribution having distinct frequency.

2 CLASSIFICATION INTO FREQUENCY COMPONENTS

2.1 Measurement of magnetic fields

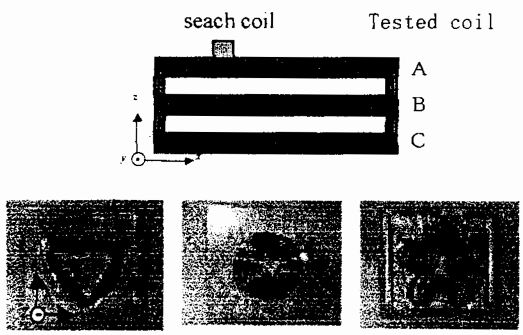
Figure 1 shows three tested coils. By piling up to z-axis, we assume them as multi-layered coil. Also, the exciting frequency was set to 10kHz(A), 30kHz (B) and 50kHz(C) respectively. We have measured magnetic fields from the upper and bottom of a 11 by 11 cm^2 current carrying coil surface. Table 2 shows the measurement distance from the each coil.

2.2 Classification of magnetic fields

According to Faraday's Law, the relationship between the induced voltage and magnetic flux is given by

$$v(t) = -N\frac{d\varphi(t)}{dt}. \quad (1)$$

Considering the multi-layered coil operating under different frequency, the measured waveform of induced voltage is deformed in Fig. 2. Under the assumption that any non-linear material is not contained, it is possible to apply Fourier transform and to classify into distinct time frequency components. Fig. 3 shows the Fourier spectrum distribution. Applying inverse Fourier transform to the dominant components, the classified magnetic fields by each frequency are obtained. Fig. 4 shows the classified magnetic fields. The upper and lower figures show magnetic fields measured from the top and bottom surface, and in these contours, white, dark and black parts denote the high, low and zero magnetic field intensities, respectively.



(a) A- layer (b) B-layer (c) C-layer

Fig.1 multi-layered coil

Table1 Basic dimension of search coil

Turn	Radius
144	0.5(cm)

Table2 The distance between each layer and measurement surface

	top	bottom
A	0.6(cm)	1.3(cm)
B	0.9(cm)	1.0(cm)
C	1.3(cm)	0.6(cm)

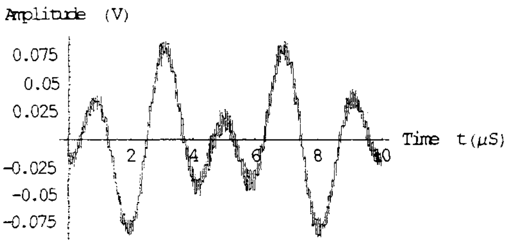


Fig.2 Measured waveform

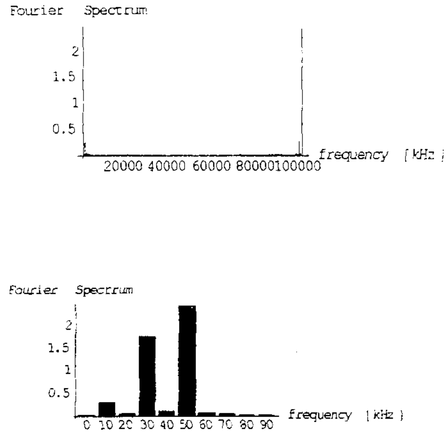
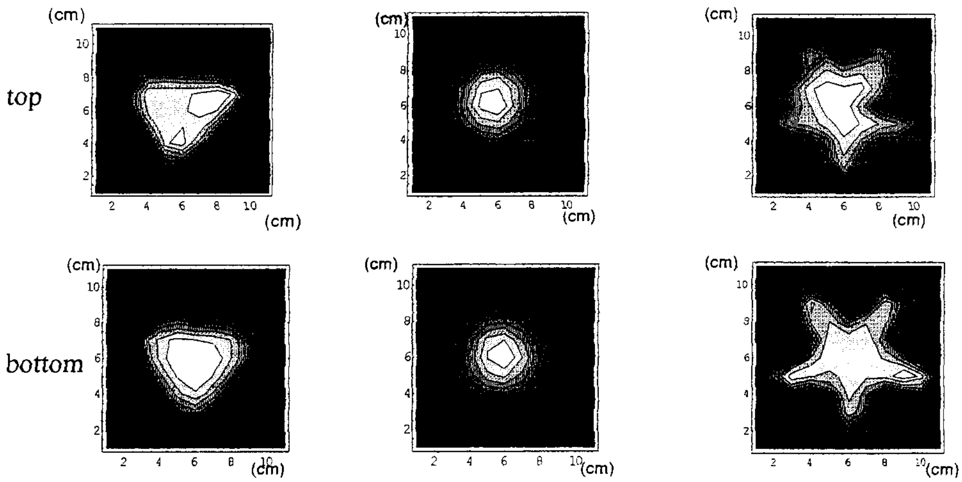


Fig.3 Fourier spectrum



A B C

Fig.4 Classified magnetic fields distribution

2.3 Loop current model

As is well known, a relationship between the currents and the magnetic fields is given as a solution of the Maxwell's equations. Fig. 5 shows a typical example of a loop current i and measured magnetic field H. As shown in Fig. 6, let us assume that the current flowing surface is divided into a large number of the loop currents [4,5]. A relationship between the loop current and magnetic field is given in terms of the elliptic integrals $E(\kappa), K(\kappa)$ as

$$H_Z = \frac{1}{2\pi}\left[\frac{z}{\sqrt{(a+r)^2+z^2}}\right]\left[\frac{a^2-r^2-z^2}{(a-r)^2+z^2}E(\kappa)+K(\kappa)\right], \tag{2}$$

where the parameters a,r,z are shown in Fig.5 and also,

$$\kappa^2 = \frac{4ra}{(r+a)^2+z^2}. \tag{3}$$

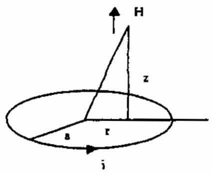


Fig.5. Relationship between the loop current i and magnetic field **H**.

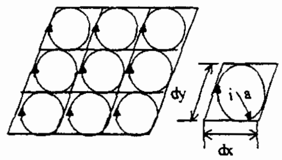


Fig.6. Subdivided loop currents model.

2.4 Current distributions by GVSPM method

Let us consider a linear system of equations

$$\mathbf{Y} = C\mathbf{X}, \tag{4}$$

where the vector Y, X are the n-th order input and m-th order output vectors, respectively. The matrix C is a n by m rectangular system matrix. In magnetic system, the vectors Y, X are the magnetic field vector with order n and the current vectors with order m, respectively. The n by m rectangular system matrix C is composed of (3). In our experiment, the number of entire given magnetic fields is 11×11. We assumed a 22×22 loop current distribution model to the target printed circuit board. Thereby, we have to solve (4) under a condition m>n. In order to solve ill-posed problem, we employed the GVSPM method[6]. On the other side, (4) can be rewritten by

$$\mathbf{Y} = \sum_{i=1}^{m} x_i \mathbf{C}_i, \tag{5}$$

$$\mathbf{X} = [x_1 \quad x_2 \quad . \quad x_m]^T, C = [\mathbf{C}_1 \quad \mathbf{C}_2 \quad . \quad \mathbf{C}_m] .$$

Further modification to (2) becomes

$$\frac{\mathbf{Y}}{|\mathbf{Y}|} = \sum_{i=1}^{m} x_i \frac{|\mathbf{C}_i|}{|\mathbf{Y}|}\frac{\mathbf{C}_i}{|\mathbf{C}_i|}$$

or

$$\mathbf{Y}' = C'\mathbf{X}'. \tag{6}$$

(6) means that the normalized input vector **Y'** is always given by a linear combination of the weighted solutions $x_1|\mathbf{C}_1|/|\mathbf{Y}|,\ x_2|\mathbf{C}_2|/|\mathbf{Y}|,\ldots,x_m|\mathbf{C}_m|/|\mathbf{Y}|$ with the normalized column vectors $\mathbf{C}_1/|\mathbf{C}_1|,\ \mathbf{C}_2/|\mathbf{C}_2|,\cdots,\mathbf{C}_m/|\mathbf{C}_m|$.

Therefore, when an angle between the input vectors of **Y** and of $C\mathbf{X}^{(k)}$ given in terms of the k-th iterative solution $\mathbf{X}^{(k)}$ is defined by

$$f\left(\mathbf{X}^{(k)}\right)=\frac{\mathbf{Y}}{|\mathbf{Y}|}\cdot\frac{C\mathbf{X}^{(k)}}{\left|C\mathbf{X}^{(k)}\right|}=\frac{\mathbf{Y}}{|\mathbf{Y}|}\cdot\frac{|\mathbf{Y}|}{|\mathbf{Y}|}\frac{C\mathbf{X}^{(k)}}{\left|C\mathbf{X}^{(k)}\right|}=\frac{\mathbf{Y}}{|\mathbf{Y}|}\cdot\frac{\sum_{i=1}^{m}x_i^{(k)}\frac{|\mathbf{C}_i|}{|\mathbf{Y}|}\frac{\mathbf{C}_i}{|\mathbf{C}_i|}}{\left|\sum_{i=1}^{m}x_i^{(k)}\frac{|\mathbf{C}_i|}{|\mathbf{Y}|}\frac{\mathbf{C}_i}{|\mathbf{C}_i|}\right|}=$$

$$=\mathbf{Y}'\cdot\frac{C'\mathbf{X}'^{(k)}}{\left|C'\mathbf{X}'^{(k)}\right|}, \tag{7}$$

so that

$$f\left(\mathbf{X}^{(k)}\right)\rightarrow 1, \tag{8}$$

means that the solution vector $\mathbf{X}^{(k)}$ satisfies the (5), i.e.,

$$\mathbf{Y}'=C'\mathbf{X}'^{(k)}. \tag{9}$$

When an initial solution vector $\mathbf{X}'^{(0)}$ is given by

$$\mathbf{X}'^{(0)}=C'^{T}\ \mathbf{Y}', \tag{10}$$

the first deviation to the normalized input vector **Y'** becomes

$$\Delta\mathbf{Y}'^{(1)}=\mathbf{Y}'-\frac{C'\mathbf{X}'^{(0)}}{\left|C'\mathbf{X}'^{(0)}\right|}. \tag{11}$$

By means of (10) and (11), the k-th iterative solution vector $\mathbf{X}'^{(k)}$ is given by

$$\mathbf{X}'^{(k)}=\mathbf{X}'^{(k-1)}+\Delta\mathbf{X}'^{(k)}=\mathbf{X}'^{(k-1)}+C'^{T}\ \Delta\mathbf{Y}'^{(k)}=\mathbf{X}'^{(k-1)}+C'^{T}\left(\mathbf{Y}'-\frac{C'\mathbf{X}'^{(k-1)}}{\left|C'\mathbf{X}'^{(k-1)}\right|}\right)$$

$$=C'^{T}\ \mathbf{Y}'+\left(I_m-\frac{C'^{T}C'}{\left|C'\mathbf{X}'^{(k-1)}\right|}\right)\mathbf{X}'^{(k-1)}, \tag{12}$$

where I_m is a m by m unit diagonal matrix.

After 100 iterations applying (12) to our problem, the current vector distributions were computed. Fig. 7 shows a contour line of the independently evaluated current distributions, and Fig.8 shows the current vectors, whose maximum magnitudes are normalized to 1.

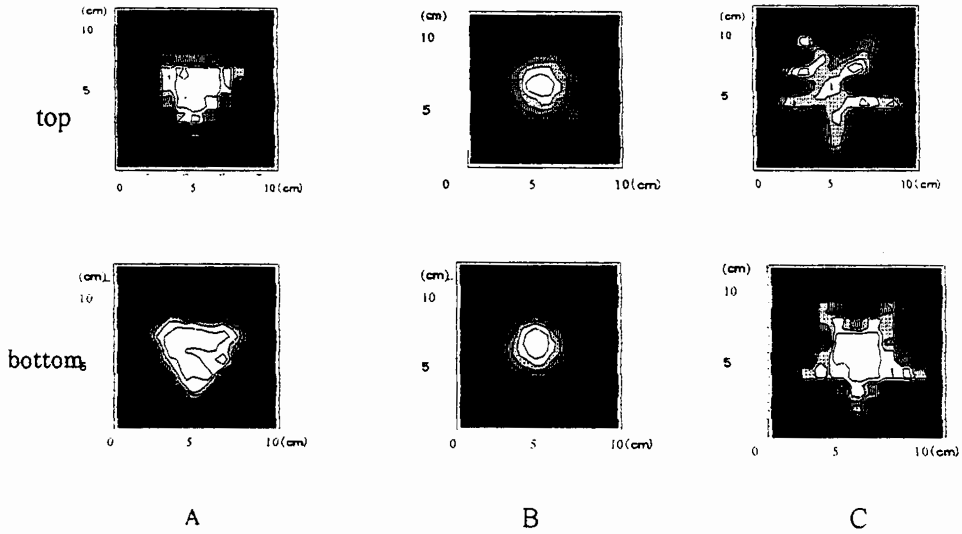


Fig.7. Evaluated current distribution contour

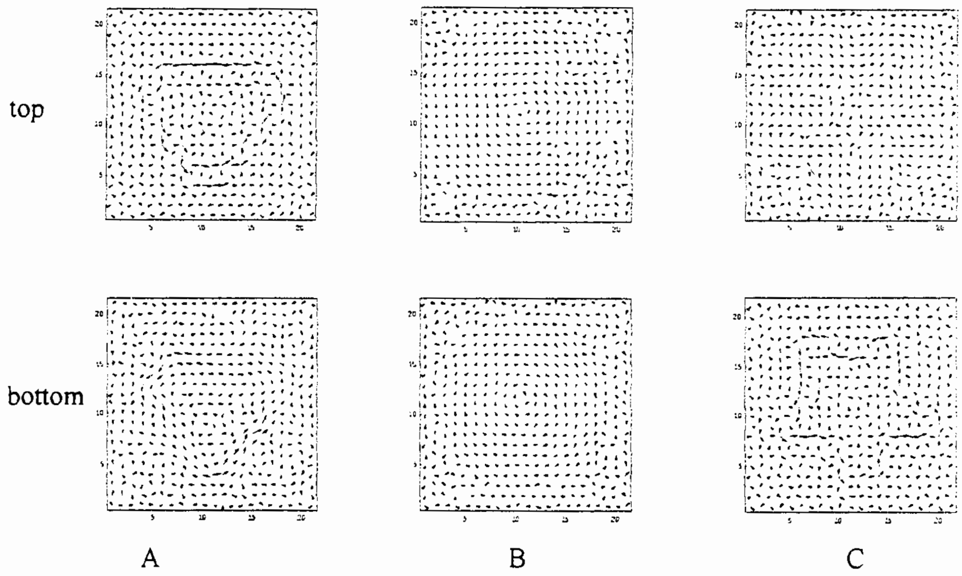
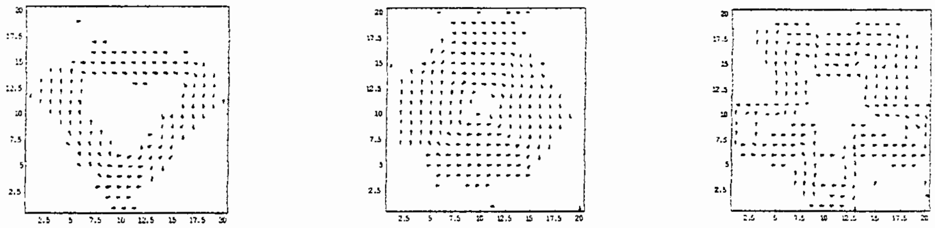


Fig.8. Evaluated current distribution vectors

2.5 Convolution strategy

Shown in Fig. 8, it is difficult to identify the current flowing paths easily. Further in order to extract the common current vector distributions among the independently computed current vectors in Fig.8, each of the current vector components is convoluted. This convolution makes it possible to extract the only dominant current vector distributions without using any threshold operation [7]. Fig. 9 shows the extracted current vectors.



A B C

Fig.9. Convolved current distribution vectors.

3 CONCLUSION

In the present paper, we have proposed a methodology to the current evaluations operating under distinct different frequency in a multi-layered-PCB. Applying the Fourier transform leads to classify the magnetic fields into distinct time frequency components. Second, employing the GVSPM method, each of the current distributions having distinct frequency is evaluated from the top and bottom magnetic field components, respectively. In order to extract the common current vector distributions, each of the current vector components has been convoluted. This yields the most reliable current distribution on a PCB. As a problem to solve, it is necessary to consider how to deal with the eddy current among the coils.

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