Applications of Electromagnetic Phenomena in Electrical and Mechanical Systems
S. Yamada, A. Nafalski and S.C. Mukhopadhyay (eds.)

A study of reconstruction algorithm in electrical impedance tomography

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ABSTRACT: This paper describes some new progress of reconstruction algorithm of Electrical Impedance Tomography, which includes Quasi-Newton, Bulirsch-Stoer extrapolation and Local Area Accelerating Convergence methods. These novel methods make it possible to improve the precision as well as computation speed dramatically in the reconstruction of Electrical Impedance Tomography. Intensive simulation demonstrates that our proposed methods are effective on reconstruction calculation of Electrical Impedance Tomography.

1 INTRODUCTION

Electrical Impedance Tomography (EIT) is a new type of medical imaging technique, which is noninvasive and inexpensive. The goal of EIT is to reconstruct the conductivity distribution by injecting electrical current on the periphery of the cross section of a human body. The reconstructed conductivity distribution will give diagnostic information about the patient's health.

One of core technologies of EIT is reconstruction algorithm. This problem is essentially reduced into solving an ill-posed inverse problem in a most efficient manner. We have to find the conductivity distribution on a cross-sectional target area from the boundary potential distribution enclosing this target area, though boundary potentials are insensitive to the variation of conductivities in the field. Fig.1 (a) shows the set conductivity distribution. Fig.1 (b) shows that the variation of potential takes relatively low sensibility to the variation of conductivities. Here the rate of change of boundary potential reaches to only 3.32% while the conductivity varies from 1 to 55.

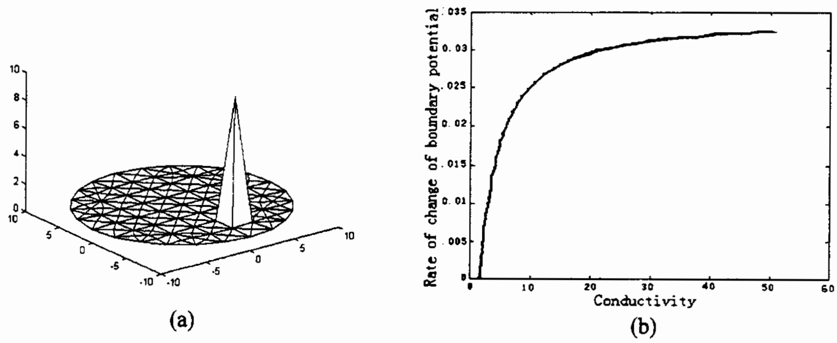


Fig.1. Subpolt (a) shows the set conductivity distribution. Subplot (b) plots the relationship between the conductivity and the variation of boundary potential.

Thus, it is obvious that a serious difficulty of EIT should be removed by solely depending on our reconstruction algorithm.

2 QUASI-NEWTON METHOD

In order to clarify the implicit relation between inner conductivities and boundary potentials, we employ Quasi-Newton method, which is one of the most effective methods to solve nonlinear equations, which is one of the most effective methods to solve nonlinear equations. Denoting B_k as Jacobian matrix at $k-th$ iteration, for this equation

$$F(X) = 0, \tag{1}$$

the iterative algorithm is shown as follow[1] [2]

$$X^{(k+1)} = X(k) - B_k^{(-1)} F(X^{(k)}), k = 0,1,..., \tag{2}$$

$$B_{k+1} = (F(X^{(k+1)}) - F(X^{(k)})) / (X^{(k+1)} - X^{(k)}), \tag{3}$$

where the superscripts k and $k+1$ denote the $k-th$ and $(k+1)-th$ iterations, respectively. As a matter of fact, φ is a function of boundary conductivity σ, that is

$$\varphi = f(\sigma). \tag{4}$$

After discretizing the governing equation of EIT, Laplace equation $-\nabla \cdot \sigma \nabla \varphi = 0$, potentials of boundary nodes, $\varphi_i, i = 1,2,..., N,$ are related by the conductivities, $\sigma_j, j = 1,2,..., E,$ as

$$\varphi_i = f_i(\sigma_1, \sigma_2, ..., \sigma_E), i = 1,2,..., N, \tag{5}$$

where E and N are the numbers of unknown conductivities σ and surface potentials φ at boundary nodes, respectively.

Consider the function shown in Eq. (1)

$$F(\sigma) = \bar{\varphi} - f(\sigma), \tag{6}$$

where $\bar{\varphi}$ is the practically measured surface potential at boundary nodes. Then we have

$$F(\sigma) = 0. \tag{7}$$

Thus, we can apply Quasi-Newton method to solve for Eq. (7). Consequently, iterative processes of Eq. (2) and Eq.(3) are reduced into

$$\sigma^{(k+1)} = \sigma^{(k)} - H_k^{(-1)} F(\sigma^{(k)}), k = 0,1,... \tag{8}$$

$$H_{k+1} = [F(\sigma^{(k+1)}) - F(\sigma^{(k)})] / (\sigma^{(k+1)} - \sigma^{(k)}), \tag{9}$$

where the Jacobian matrix B_k in Eq. (2) is corresponding to the reconstruction matrix H_k. Further, we impose the following constraint:

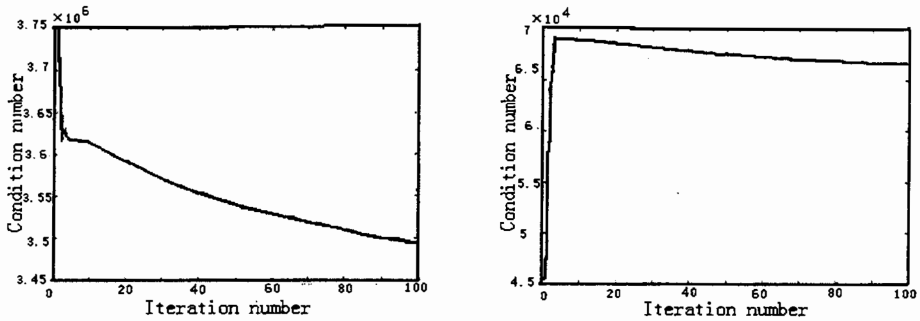
$$\sigma^{(k+1)} - \sigma^{(k)} = c, \tag{10}$$

In Eq. (10), c is an experimentally determined numerical constant [2]. As an initial conductivity, set explicit conductivities σ_{set} in whole computation field

$$F(\sigma^{(k)}) = \varphi_{set} - f(\sigma^{(k)}), \tag{11}$$

where φ_{set} is the ideal surface potential obtained by solving the EIT governing equation with conductivity σ_{set}. According to the iterations, reaching zero by $F(\sigma^{(k)})$ in Eq.(11) means that $\sigma^{(k)}$ is also reaching σ_{set}.

In practical reconstruction processes, the reconstruction matrix H_k becomes ill-posed[3]. This difficulty can be somehow removed by Tikhonov regularization. Fig.2 shows the relationship between the iteration number and the condition number while reconstructing the conductivity distribution shown in Fig.1(a). The condition number reveals ill-posed nature of the matrix. Figs.2 (a) and (b) are the iteration number vs. the condition numbers before and after using Tikhonov regularization respectively. The result suggests that Tikhonov regularization can help to overcome the difficulty of reconstruction calculation when employing Quasi-Newton method. Thus, the Quasi-Newton method can be usefully applicable to our reconstruction processes.



(a) (b)

Fig.2. Relationship between iteration number and condition number of reconstruction matrix. (a) Quasi-Newton method without Tikhonov regularization (b) Quasi-Newton method with Tikhonov regularization

3 BULIRSCH-STOER EXTRAPOLATION METHOD

Even though Quasi-Newton method is a powerful method to solve nonlinear equations, it suffers from a high computational cost of calculating the reconstruction matrix H employing the fine mesh system. To overcome this difficulty, we employ Bulirsch-Stoer extrapolation method, which makes it possible to improve the reconstruction velocity [4].

The key of extrapolation method is based on the error analysis of the calculated object. Extrapolation method enables us to accelerate convergence by increasing the error-order piece-by-piece. The Bulirsch-Stoer extrapolation method is applied to Eq. (8) by

$$\sigma_{n+1} = T_n^{(0)} - H_n^{(-1)} F(T_n^{(0)}), n = 0,1,..., \tag{12}$$

where T is extrapolation sequence, and its iterative formulas are as follows[4]:

$$T_0^{(i)} = \sigma_i, i = 0,1,..., \tag{13}$$

$$T_m^{(i)} = T_{m-1}^{(i+1)} + (T_{m-1}^{(i+1)} - T_{m-1}^{(i)})/(1/b^m - 1),$$
$$i = 0,1,..., \; m = 1,2,..., \; 0 < b < 1. \tag{14}$$

To reduce the workload in practical computation, Eqs.(13) and (14) are carried out by[4]

$$\begin{array}{l} T_0^{(0)} \\ \to T_0^{(1)} \to T_1^{(0)} \\ \to T_0^{(2)} \to T_1^{(1)} \to T_2^{(0)} \\ \to T_0^{(3)} \to T_1^{(2)} \to T_2^{(1)} \to T_3^{(0)} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \to T_0^{(i+1)} \to T_1^{(i)} \to T_2^{(i-1)} \to T_3^{(i-2)} \to \cdots \to T_{i+1}^{(0)} \end{array} \tag{15}$$

In processes Eqs.(15), every column decreases the convergence pace, and every row increases the error-order. Thus, the requirement for sufficient small error can be realized quickly. In reconstruction calculation presented in this paper, the reconstruction matrix H is a 276 by 125 rectangular matrix, T is a 125 by 1 array matrix, F is a 276 by 1 array matrix, b is a constant chosen by the experiment, and means convergence pace.

Intensive simulating calculation experiments have suggested that our methodology is one of the most effective extrapolations for EIT.

4 LOCAL AREA ACCELERATING CONVERGENCE (LAAC) METHOD

In order to reduce the computation time as well as accumulated error essentially accompanying with numerical calculation, we propose an experiential method named Local Area Accelerating Convergence (LAAC) method, which dramatically reduces the accumulated calculation error, and also improves computation speed greatly.

In our simulating reconstruction calculations with Quasi-Newton method along with Tikhonov regularization as well as Bulirsch-Stoer extrapolation, we find that the conductivities of disturbed nodes always are closer to original values with the increasing of iterative times. On the other hand, the conductivities of ordinary nodes are closer to original values with the increasing of iterative times also, but when they closed enough, if we continue calculating, accumulative error will become greater and greater

In our new method, when the conductivity of node changes less with the increasing of iterative times, we stop calculating on this node and let the current value be the final value. So, the number of unknown conductivities of nodes becomes less and less, and calculation speed improves greatly. It is why we call this new method as Local Area Accelerating Convergence.

LAAC method base on prior known information of the conductivity distribution, our reconstruction calculation experiments prove that it is definite effective.

5 RESULTS

In our simulations, a model target cross-sectional area is subdivided into 256 triangular elements with 125 nodes, and also it equips 24 boundary electrodes. In Figs.3 and.4, (a) and (b) show the original conductivity distribution, (c) and (d) show the reconstructed conductivity distribution. In these figures, the dark and light denote the conductivities taking high and low values, respectively. In Figs.3 and 4, (a) and (c) show the 2-D distribution of conductivity in target region, (b) and (d) show the magnitude of conductivity.

Comparison the original conductivity distribution with those of reconstructed one in Fig.3 suggests that reconstructed conductivities of disturbed nodes take the values around 50 percent of original ones. Fig.4 shows the other example having six disturbed nodes. In this case, there are two pair of adjacent disturbed nodes. Obviously, the reconstruction image reflects on the relative difference of conductivities. All reconstructed images can reflect the original images well.

6 CONCLUSIONS

Since the inverse problem in EIT never have unique solution, each slight improvement leads to a great step of EIT development. In this paper, all reconstructed images demonstrate that our novel approaches, which are Quasi-Newton, Bulirsch-Stoer extrapolation and Local Area Accelerating Convergence methods, are feasible in reconstruction calculation of EIT.

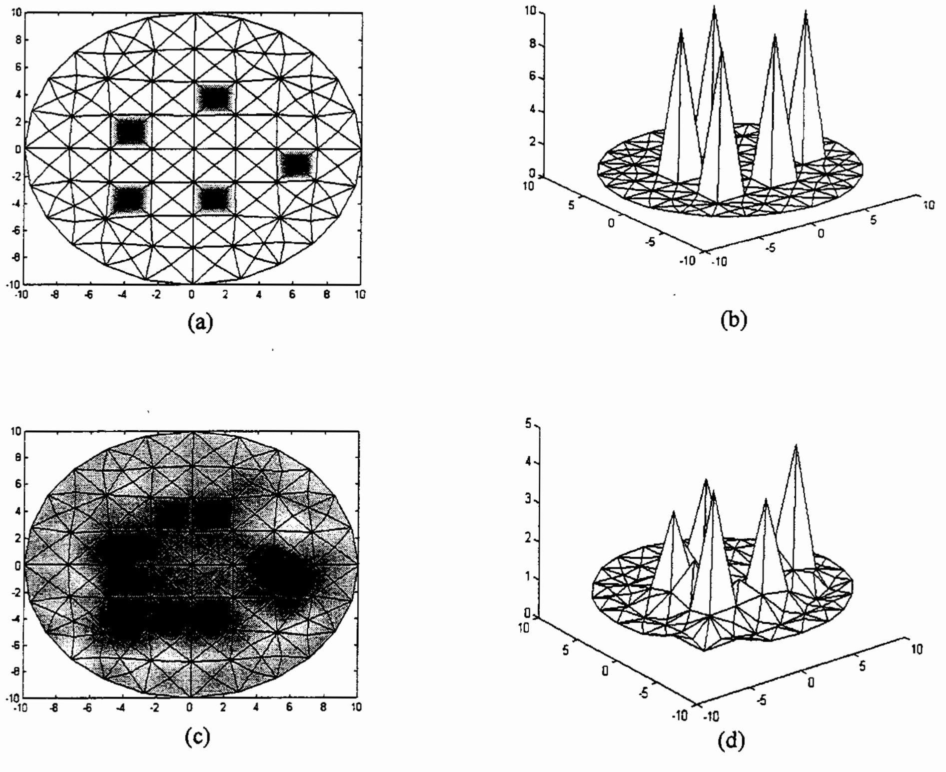


Fig.3. The original and reconstructed conductivity distributions. (a) Original conductivity distribution represented by 2-D gray map, (b) original conductivity distribution represented by 3-D map, (c) reconstructed conductivity distribution represented by 2-D gray map, (d) reconstructed conductivity distribution represented by 3-D map.

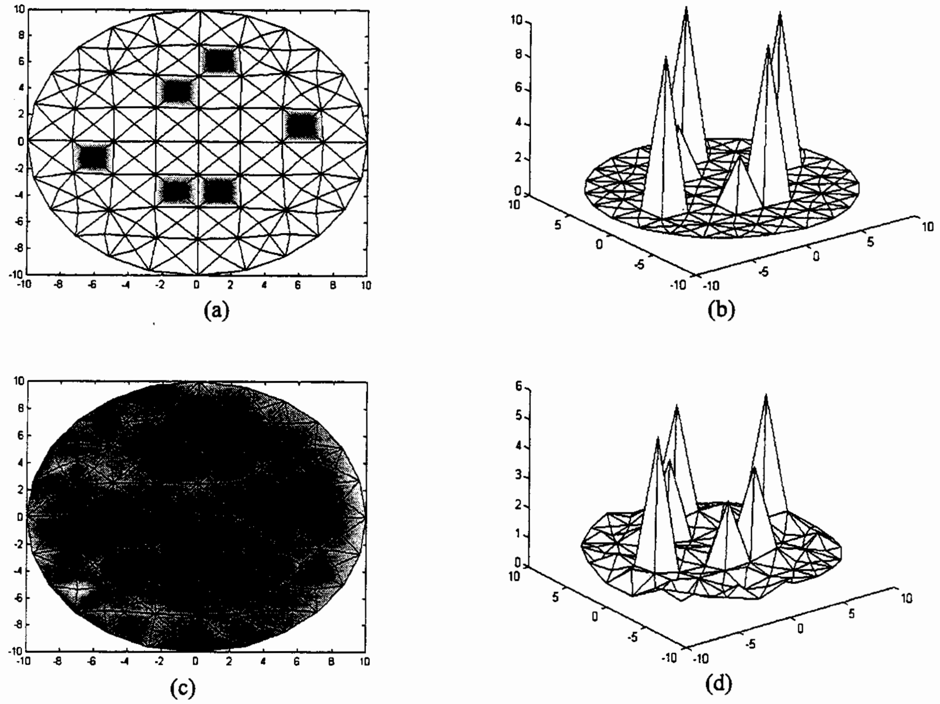


Fig.4. The original and reconstructed conductivity distributions. (a)Original conductivity distribution represented by 2-D gray map, (b) original conductivity distribution represented by 3-D map, (c) reconstructed conductivity distribution represented by 2-D gray map, (d) reconstructed conductivity distribution represented by 3-D map.

7 REFERENCES

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