Applied Electromagnetics III

Weighted Inverse Approach for Searching of the Current Distribution from the Locally Measured Magnetic Fields

Shusuke Mabuchi, Seiji Hayano, Yoshifuru Saito
(c/o Y.Saito Lab., Hosei University, Koganei, Tokyo 184-8584, Japan)

Abstract. Previously, we have proposed the weighted inverse method in order to solve an ill-posed system of equations. By means of this method, we independently evaluate the 2D and locally 1D current vector distributions from the locally measured magnetic fields. After that the convolution operations among the obtained current vectors by 2D and locally 1D yield the precise current distributions.

1. Introduction

In order to check up the fault points and electromagnetic compatibility of the electronic devices, evaluation of a 2D current vector distribution from the locally measured magnetic fields is one of the deterministic methodologies. Evaluation of the current distributions from the locally measured magnetic fields is essentially reduced into solving for an ill-posed system of equations. Previously, we have proposed the weighted inverse method, which was a generalization of least norm method [1].

The weighted inverse matrix is one of the generalizations of least norm method, but assumes that the solution of weighted inverse matrix method can be expanded in functional series. Thereby, examination of convergence property of the series makes it possible to the validity of solution.

In the present paper, this weighted inverse method is independently applied to the evaluations of 2D- and locally-1D current vector distributions. After that, it is shown that the convolutions among the 2D- and locally-1D current vectors extract the detailed current distributions.

2. Current Vectors Searching by Weighted Inverse Approach

2.1 Weighted Inverse Matrix Method

Most of the inverse problems are reduced into solving for a linear system of equations, which is ill posed, i.e. the number of unknowns are much larger than those of givens. Thereby, it is difficult to solve such ill posed system of equations without any constraints.

A generalized inverse matrix approach is one of the ways to solve the ill posed linear system of equations. Least norm method is a typical generalized inverse approach. This method always gives a solution. But, the least norm solution hardly corresponds to a physically existing solution.

Our weighted inverse matrix (WIM) method is one of the generalized inverse approaches. To introduce the WIM method, let us consider the simplest ill-posed equation (1). It is composed of only one equation having constants a,b,c, but the number of unknowns x and y is two.

$$ax + by = c . \quad (1)$$

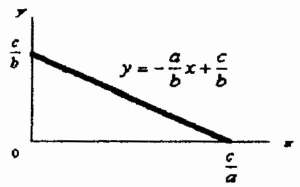


Fig. 1. Solution line of (1).

In (1), the value x=0 and y=0 give the solutions $y=c/b$, $x=c/a$, respectively. This reveals that infinite number of solutions exist on a straight line in Fig.1. Thus, we have a solution line given by

$$y = -\frac{a}{b}x + \frac{c}{b}. \quad (2)$$

According (2), fixing the value of x determine the corresponding value of y, or vice versa. Thereby, single equation gives single unique solution, i.e., number of solutions, equivalent to those of equations, is uniquely determined.

Denoting w_1, w_2 to be the weights of the solution S_0, we assume that the solutions x and y are represented by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} S_0. \quad (3)$$

Substituting (3) into (1) in terms of the matrix form yields a following relationship:

$$[a \quad b]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} S_0 = c. \quad (4)$$

Thus, by means of (4), it is possible to obtain the solution vector of (1) as

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} S_0 \\ &= [w_1 a + w_2 b]^{-1} c \end{aligned} \quad (5)$$

The weighted inverse solution of (1) is given by (5). If an ill-posed system of equations is expressed by

$$\mathbf{Y} = C\mathbf{X}. \quad (6)$$

then the solution vector $\mathbf{X}$ of (6) can be represented in a form of

$$\mathbf{X} = W\mathbf{S}. \quad (7)$$

where $\mathbf{X}$, $\mathbf{Y}$ and C are the solution vector with order of m, given input vector with order n, and n by m system matrix, respectively.

Since the solution vector $\mathbf{X}$ is a function of solution space α, then the solution $s(\alpha)$ can be represented in a series form:

$$s(\alpha) = s_0 + \alpha s_1 + \alpha^2 s_2 + \cdots + \alpha^{n-1} s_{n-1}. \quad (8)$$

Discretization of (8) with solution space step-width $\Delta\alpha$ gives the solution vector $\mathbf{X}$:

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & \Delta\alpha & \Delta\alpha^2 & \cdots & \Delta\alpha^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & (n-1)\Delta\alpha & [(n-1)\Delta\alpha]^2 & \cdots & [(n-1)\Delta\alpha]^{n-1} \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ \vdots \\ s_{n-1} \end{pmatrix} = W\mathbf{S}. \quad (9)$$

Thus, the formal weighted inverse solution of (6) is given by

$$\mathbf{X} = W\mathbf{S} = W[CW]^{-1}\mathbf{Y}. \quad (10)$$

Examination of the elements in vector **S** reveals the validity of solution vector **X**. The solution vector **X** is an analytical function when the coefficients $s_0, s_1,.., s_{n-1}$ are converged to be small in value [2].

2.2 Current Vectors Estimation by 2D WIM

In order to verify our WIM method, we computed the current vectors from the locally measured magnetic field. Fig.2 shows a tested coil.

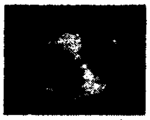


Fig. 2. Tested coil.

The perpendicular magnetic fields at the 5×5=25 points to the coil surface are measured above the coil. Fig.3 shows the measured magnetic field, where the white and black tones are corresponding to the positive and negative values, respectively.

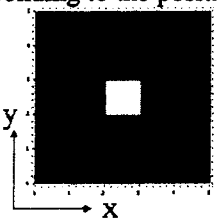


Fig. 3. Measured magnetic field.

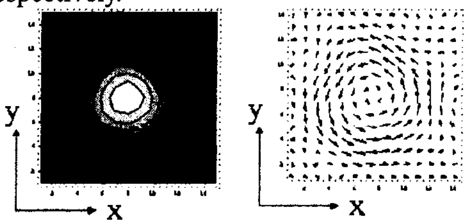


Fig. 4. Computed 2D loop current (left) and vector (right) distributions.

Fig.4 shows the computed 2D loop current and vector distributions by means of the WIM method along with the loop current model [2]. In this figure, it is obvious that the computed 2D loop current and vector distributions well reflect the shape of tested coil shown in Fig.2. Figs.5 and 6 show the other measured magnetic field and current distribution, respectively. Fig.7 shows the values of coefficients $s_0, s_1,.., s_{24}$ in the vector S in (9). Observation of the convergence properties in Fig.7 verifies the validity of the current vectors in Figs.4 and 6.

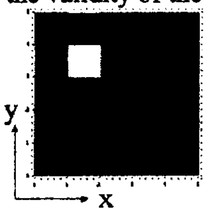


Fig. 5. Measured magnetic field.

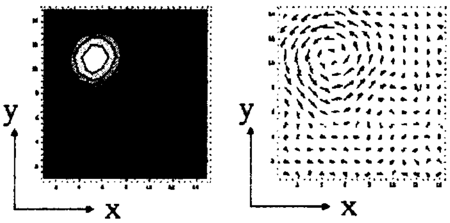
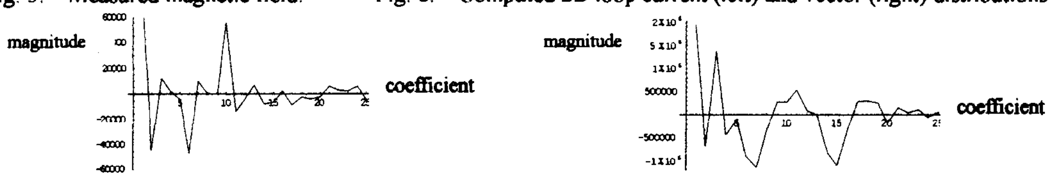


Fig. 6. Computed 2D loop current (left) and vector (right) distributions.



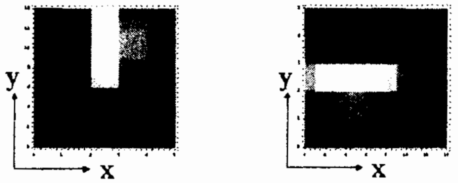
Coil located at center

Coil located at left up

Fig. 7. Convergence properties of the WIM solutions.

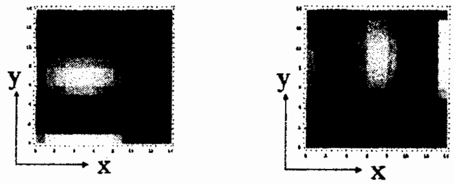
2.3 Current Vectors Estimation by 1D WIM Method

Fig.8 shows the measured magnetic fields in the directions of x- and y- axes, respectively. We computed the y- and x- directed current vectors from the x- and y- directed magnetic field distributions, respectively. Fig.9 shows the computed x- and y- directed current vectors by means of WIM method along with the line current model. In Figs. 8 and 9, white and black tones denote the positive and negative values, respectively.



x- component　　　　y- component

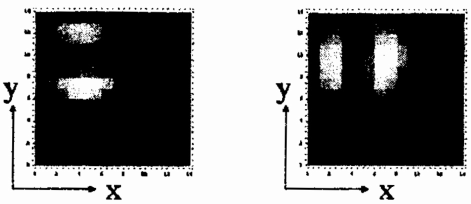
Fig. 8. Measured magnetic field components in the x-, y-directions



x- component　　　　y- component

Fig. 9. x- and y- directed current components computed from the magnetic fields in Fig.8.

After separating x- and y- directed current vectors from 2D current vector of Figs. 6, convolution among the 2D and 1D WIM current vectors yields the x- and y- directed current vector components shown in Fig.10. Vector composition of the x- and y- directed current vectors yields the composite current distribution as shown in Fig.11. Comparison of the current distributions in Figs. 6 and 11 reveals that the current distribution in Fig. 11 reflects the precise shape of tested coil in Fig.2.



x- component　　　　y- component

Fig. 10. Convolved current components.

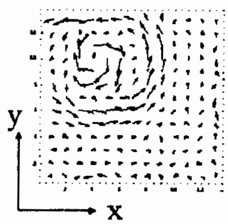


Fig. 11. Composite 2D current distribution.

3. Conclusion

As shown above, this paper has introduced the WIM method in order to obtain the physically existing solution from an ill-posed linear system of equations. Furthermore, this WIM method has been independently applied to the evaluations of 2D- and 1D current vector distributions from the locally measured magnetic field. Also it has been clarified that convolutions among the 2D and 1D WIM current vectors extract the detailed current distributions well reflecting the coil shape.

References

[1] K.Nakajima, S.Hayano and Y.Saito, Application of Wavelets Analysis to Magnetic Field Source Searching. INVERSE PROBLEMS IN ENGINEERING MECHANICS, Elsevier Pub. Co., Amsterdam, (1998) 477-484.

[2] Y.Saito, Introduction to electromagnetic inverse problems, Lecture note of Y.Saito.Lab. Graduate School of Engineering, Hosei University.