ANALYSIS OF TURBULENT STRUCTURE USING DISCRETE WAVELET TRANSFORM

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ABSTRACT

To evaluate coherent structures in the dimension of time and scale, a new procedure based on the multiresolution analysis and multiresolution auto-correlation analysis is developed in this paper. By analyzing u- and v-components of fluctuating velocity, the coherent structure and its scales can be identified when larger amplitude fluctuation and stronger auto-correlation are appeared at same wavelet level. For a turbulent jet at position of x/d=6, the larger coherent structures with frequency 39*Hz* can be deduced around times 0.29, 0.53, 0.6 and 0.67s. This also represent the passing of eddies through the shear layer and concentration of the energy of the flow at these instants.

Key Words: Discrete Wavelet Transform, Eddy, Jet, Multiresolution Correlation Analysis, Turbulence

INTRODUCTION

There exist many different elementary structures in turbulent flows. One of important structure is called coherent structure, and it is usually defined as *a region of the flow over which at least one of fundamental flow variable exhibits significant correlation with itself or with another variable over a range of space and/or time that is significantly larger than the smallest local scales of the flow* (Robinson, 1991)[(1)]. Coherent structures are known to exist in a turbulent jet, and be responsible for most of the substantial mass, heat and momentum transport. The coherent structures are condensation of the vorticity field into organized structure and concentrate much of the energy of flow. Characterizing its physics through observations and its behavior under varying conditions is still a topic of considerable continued study.

Until now the conventional statistical methods and visualization techniques are well-established usual techniques for gaining information regarding the nature of coherent structure. However, coherent structure is characterized by unsteady and localized structure of multiple spatial scales, and the coherent structure in both time and scale spaces has not yet been clarified and explained. Neither the Fourier analysis nor the traditional correlation method gives us sufficient information owing to the non-local nature of the Fourier analysis.

In recent decade, there has been growing interest in the wavelet analysis of turbulent signal, which can combine time-space and frequency-space analyses to produce a potentially more revealing picture of time-frequency localization of turbulent structure. The continuous wavelet transform has been proposed to track coherent structures (Li, 1998a and 1998b) [(2), (3)] in terms of time and scale. They offered the potentials extracting new information from turbulent flows. The coefficients of continuous wavelet transform are known to extract the characterization of local regularity, but it is unable to reconstruct the original function because the mother wavelet function is non-orthogonal function. In the signal processing, it is importance to reconstruct the original signal from wavelet composition and to study multiresolution signal in the range of various scales.

The discrete wavelet transform allows an orthogonal projection on a minimal number of independent modes and is invertible and in fact orthogonal inverse transform. Such analysis is known as a multiresolution representation and might be used to compute or model turbulent flow dynamics. Li et al. (1999)[(4)] applied the two-dimensional orthogonal wavelets to turbulent images, and extracted the multiresolution turbulent structures. However, few investigations concerned the extraction of coherent structure in terms of time and scale.

To identify the coherent structures in the dimension of time and scale, we proposed a new procedure based on the discrete wavelet transforms in this paper.

DISCRETE WAVELET TRANSFORM

The discrete wavelet transform is a transformation of information from a fine scale to a coarser scale by extracting information that describes the fine scale variability (the detail coefficients or wavelet coefficients) and the coarser scale smoothness (the smooth coefficients or mother-function coefficients) according to:

$$\{\boldsymbol{S}_{\boldsymbol{j}}\} = [\boldsymbol{H}]\{\boldsymbol{S}_{\boldsymbol{j}+1}\} \quad ; \qquad \{\boldsymbol{D}_{\boldsymbol{j}}\} = [\boldsymbol{G}]\{\boldsymbol{S}_{\boldsymbol{j}+1}\} \qquad (1)$$

where $\boldsymbol{S}$ represents mother-function coefficients, $\boldsymbol{D}$ represents wavelet coefficients, j is the wavelet level, and $\boldsymbol{H}$ and $\boldsymbol{G}$ are the convolution matrices based on the wavelet basis function. High values of j signify finer scales of information.

The inverse discrete wavelet transform is similarly implemented via a recursive recombination of the smooth and detail information from the coarsest to finest wavelet level (scale):

$$\{\boldsymbol{S}_{\boldsymbol{j}+1}\} = [\boldsymbol{H}]^{\boldsymbol{T}}\{\boldsymbol{S}_{\boldsymbol{j}}\} + [\boldsymbol{G}]^{\boldsymbol{T}}\{\boldsymbol{D}_{\boldsymbol{j}}\} \qquad (2)$$

where $\boldsymbol{H}^{\mathrm{T}}$ and $\boldsymbol{G}^{\mathrm{T}}$ indicate the transpose of $\boldsymbol{H}$ and $\boldsymbol{G}$ matrices, respectively.

The matrices $\boldsymbol{H}$ and $\boldsymbol{G}$ are created from the coefficients of the basis functions, and represent the convolution of the basis function with the data. In this study, Daubechies wavelet with orders 20 is used as orthonormal wavelets, since a high order wavelet base have good frequency localization that in turn increases the energy compaction.

MULTIRESOLUTION ANALYSIS

The goal of the multiresolution analysis is to get a representation of a signal that is written in a parsimonious manner as a sum of its essential components. Multiresolution algorithm process fewer data by selecting the relevant details that are necessary to perform a particular recognition task. That is, a parsimonious representation of a signal will preserve the interesting features of the original signal, but will express the function in terms of a relatively small set of coefficients. Thus overcoming limitations of the continuous wavelet transform that cannot reconstruct the original signal. Coarse to fine searches processes first a low-resolution signal and zoom selectively into finer scales information.

In this study, the procedure of this multiresolution analysis can be summarized in two steps:

(1) Wavelet coefficients of a signal are computed based on the discrete wavelet transform of Eq.(1).
(2) Inverse wavelet transform of Eq.(2) is applied to wavelet coefficients at each wavelet level, and components of signal are obtained at each level or scale.

Of course, a sum of these essential components of signal can recover the original signal.

MULTIRESOLUTION AUTO-CORRELATION METHOD

In identifying the self-similarity structure of signals and its evolution in time, the auto-correlation analysis is most used. A difficulty with the conventional auto-correlation method, however, is that the auto-correlation function only provides information about the self-similarity behavior in terms of time delay but no information about correlation behaviors in scale space due to lack of scale resolution. Recently, Li (1998) proposed wavelet auto-correlation analysis based on the continuous wavelet transform. The wavelet auto-correlation analysis provides the unique capability for decomposing the correlation of arbitrary signals over a two-dimensional time delay-scale plane. In analogy with the wavelet auto-correlation, in this paper we first unfold a signal into its two-dimensional time-scale planes using multiresolution analysis. Then we use signal components to define an auto-correlation function at each wavelet level, called the *multiresolution auto-correlation function*.

In order to detect the coherent structures that dominate the turbulent flow in terms of energy concentration, the following procedure is proposed.

(1) Compute the components of fluctuating velocity at each level using multiresolution analysis.
(2) Compute the multiresolution auto-correlation coefficients at each level from the components of fluctuating velocity.
(3) The energy-concentrating feature i.e. coherent structure exists when large amplitude fluctuation and stronger auto-correlation are appeared at same level.

EXPERIMENTAL APPARATUS

A definition sketch of the plane jet is shown in Fig.1, where x is the streamwise coordinate, y is the lateral coordinate. The jet is generated by a blower-type wind tunnel having flow-straightening elements, screens, settling length and a 24:1 contraction leading to a rectangular nozzle of 300x20*mm* (the nozzle width $2b_0$ is 20*mm*). The measurements were carried out at a constant Reynolds number (based upon exit mean velocity, U_0, and nozzle width, $2b_0$) of approximately R_e=5000 which corresponds to an exit velocity of U_0=5*m/s*. The measured exit turbulent intensity on the jet centerline is less than 0.04%. The velocity components of u and v are measured simultaneously using an X type hot wire probe located in the (x, y)-plane. The measurements in this investigation are taken on the centerline and in the shear layer, and are shown in Fig.1. The recording frequency of data is 10*kHz*, and the number of sampled data of each signal was 15000.

RESULTS AND DISCUSSION

Using the multiresolution analysis the u-component of fluctuating velocity in the shear layer of y/b_0=1.0 at x/d=6 is decomposed in range of wavelet levels 1~8,

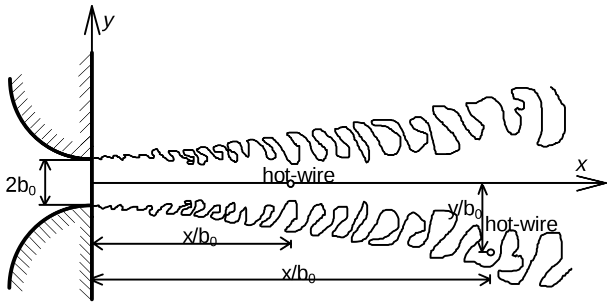

Fig.1 Sketch of the experimental configuration

which correspond to the central frequency range f=9.5~2500*Hz*, and the components of fluctuating velocity are shown in Fig.2(a). The original fluctuating velocity is also plotted in this figure. This figure exhibits the time behavior of the fluctuating velocity within different scale bands, and gives their contribution to the total turbulent energy. The smaller amplitude of velocity fluctuating at level 1 indicates that the turbulent motion with this scale don't exist. However, the large amplitude fluctuating can be clearly observed at levels 2~5, where corresponds to the central frequency to 19.5~156.3*Hz*. This indicates that the velocity components at this scale range contain almost all of the turbulent energy. Among these levels, the largest amplitude fluctuating occurs at level 3. At other levels (from levels 6 to 8) the small amplitude of velocity fluctuating appears again, which corresponds to small-scale turbulent motions.

Figure 2(b) shows the multiresolution analysis of the v-component of fluctuating velocity at same position. The large amplitude fluctuating can be clearly observed at levels 3~6 and this implies that most of turbulent energy concentrated at this scale range. Here levels 3~6 corresponds to the central frequency to 39~312.5*Hz*.

The multiresolution auto-correlation coefficients of the u- and v-component of fluctuating velocity at each level are shown in Fig.3. The conventional auto-correlation coefficients are also plotted in this figure. As time delay increases, the conventional auto-correlation coefficient decreases to zero, and it is difficulty to obtain information on the self-similarity structures. The distribution of multiresolution auto-correlation coefficients shows a map of apparent multi-scale self-similarity structures. In the range of smaller-scale i.e. from levels 5 to 8, the multiresolution auto-correlation coefficients become weak with increasing time delay. This appears to show no organized structures at higher frequencies. Although larger amplitude of auto-correlation coefficients occurs at levels 1 and 2, no coherent structures exist at large-scale, because components of fluctuating velocity exhibit smaller amplitude in multiresolution analysis of Fig.2. As increasing level or decreasing scale the amplitude of auto-correlation coefficients decreases in the range of levels 3~5. This implies the organized structures disappear. The periodic large peaks can be clearly observed at level 3, which corresponds to the central frequency to 39*Hz*, and indicate the apparent coherent structure or dominant structure. Comparing Fig.3 (a)

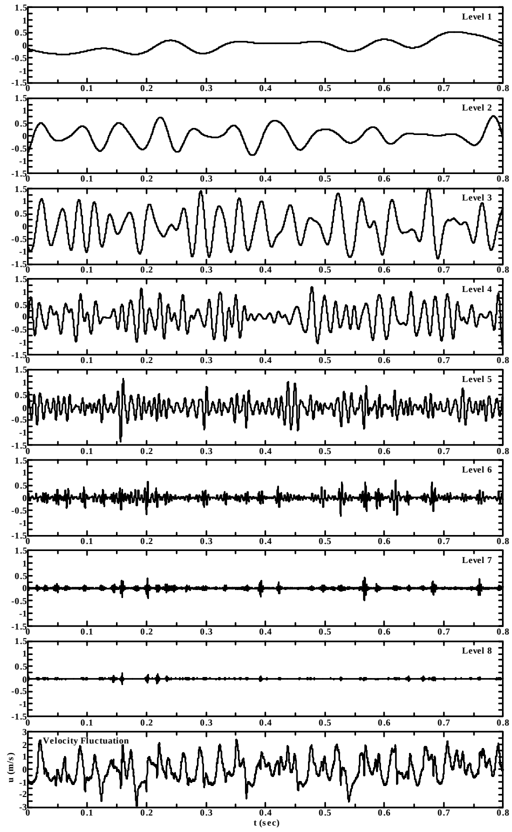

Fig.2 (a) Decomposition of u-fluctuating velocity at *x/d*=6 and *y/b_0*=1.0 based on multiresolution analysis

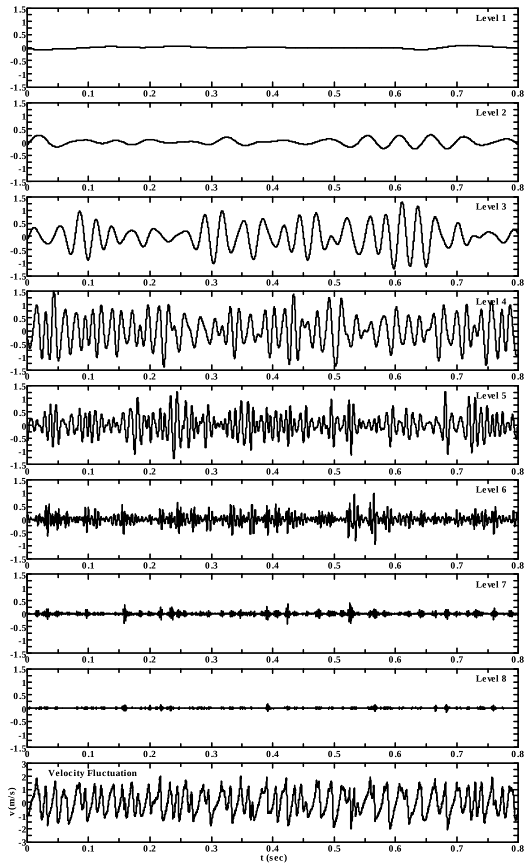

Fig.2 (b) Decomposition of v-fluctuating velocity at *x/d*=6 and *y/b_0*=1.0 based on multiresolution analysis

with Fig.3 (b) at level 3, it can be observed that the distribution of auto-correlation coefficients is similarity and the positions of peaks in Fig.3 (a) correspond to that of Fig.3(b) at the same time delay.

After determining the scale of coherent structures, it is important to analyze the coherent structures responding to the changing the time. The results of Yule (1978) [5] have shown that when an eddy passed, the local fluctuating velocity exhibits unusually large positive and negative peaks. It can say that the positive or positive peaks of the velocity fluctuation at level 3 correspond to the passing of the coherent structures or organized eddy motions. It is evident that the larger peaks occur around times 0.29, 0.53, 0.6 and 0.67s in Fig.2, which correspond to the larger coherent structures. These peaks also imply the passing of dominant eddies through the shear layer. However, these features could not have been extract using the conventional Fourier based analysis.

SUMMARY

In this paper the coherent structures of a plane turbulent jet are evaluated in the dimension of time and scale using the multiresolution analysis and multiresolution auto-correlation analysis. The following results can be summarized.

(1) From multiresolution analysis and multiresolution auto-correlation analysis, the coherent structure can be identified when larger amplitude fluctuation and stronger auto-correlation are appeared at same level.

(2) For analyzing u- and v-components of fluctuating velocity at *x/d*=6 and y/b_0=1.0, the larger coherent structures with frequency 39*Hz* can be deduced around times 0.29, 0.53, 0.6 and 0.67s. This also represent the passing of eddies through the shear layer and concentration of the energy of the flow at these instants.

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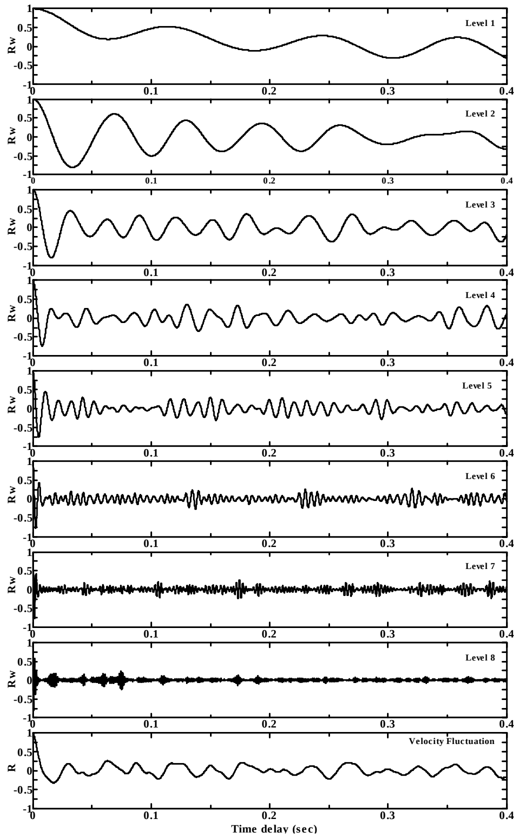

Fig.3 (a) Multiresolution auto-correlation coefficients of u-fluctuating velocity at *x/d*=6 and y/b_0=1.0

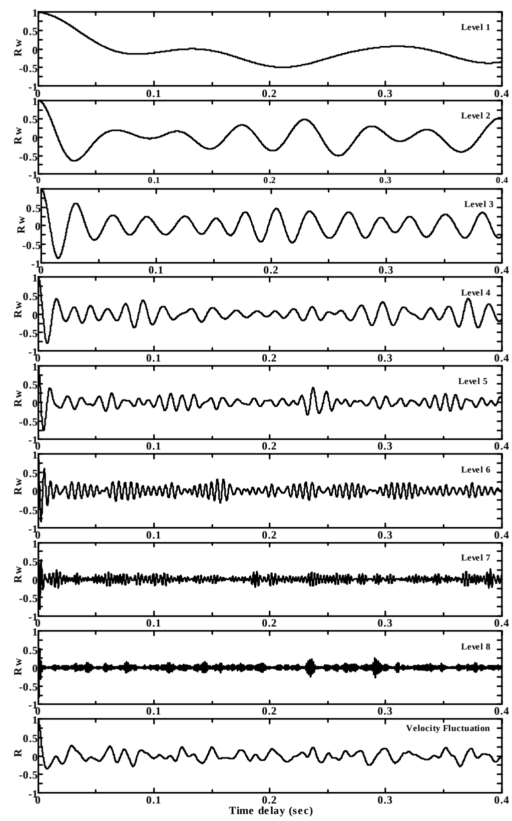

Fig.3 (b) Multiresolution auto-correlation coefficients of v-fluctuating velocity at *x/d*=6 and y/b_0=1.0