A Wavelet Transform Approach to Inverse Problems of Vandermonde Type Systems

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***Abstract* -- Inverse Problems of Vandermonde type systems have been solved using the discrete wavelet transform. The inverse matrices of the transformed subsystems were calculated, thereby locating the largest well-conditioned submatrix. The reduced system was solved and the solution was inversely transformed. Results were compared between two different wavelet basis functions, indicating that Daubechies-4 wavelets lead to much more accurate solutions than Haar wavelets. Three simple techniques for eliminating another systematic noise are also proposed to further improve the accuracy of the final solution.**

I. INTRODUCTION

In inverse problems, a Vandermonde type system matrix appears when the length of an unknown source vector equals the length of a measured field vector [1]. A possible way to solve this system is inversion of the system matrix; however, its determinant rapidly underflows for finer discretization causing this method to be impractical [1]. On the other hand, a new approach to Vandermonde type systems was proposed by Saito [2], where the Haar wavelet transform was utilized to overcome the ill-conditioned problem. In this paper, we first introduce the original concept described in [2], then show our recent improvements using a different wavelet basis function and several other techniques.

II. PRINCIPLE

To solve a system matrix equation $A x = b$, we first wavelet-transform the matrix A and the right-hand side b by $A' = W A W^T$ and $b' = W b$ where W denotes the one-dimensional wavelet transform matrix [3], then solve $A' x' = b'$. Note that wavelet transform requires that the sizes of the matrix and the vector are power of two.

In ill-conditioned Vandermonde type systems, the inverse matrices of both A and A' are not available. Besides, it is practically true that larger-magnitude components of the A' matrix are positioned around a corner while smaller-magnitude components are placed elsewhere. Therefore, by calculating the condition of the transformed system submatrices, we can locate the largest well-conditioned subsystem A''. Similarly, the b' vector has larger-magnitude components around the first half area while leaving smaller components elsewhere.

Considering the above things, let us reduce the system to the well-conditioned system $A'' x'' = b''$ where A'', x'', and b'' are the submatrix or the subvectors of A', x', and b'.

The solution of this equation is mathematically denoted by $x'' = A''^{-1} b''$. To reconstruct the wavelet space solution x' having a longer vector length, we simply add zero components to the end of the shorter solution vector x''. Finally, we transform to the solution by the inverse wavelet transform, i.e. $x = W^T x'$. More details of the wavelet transformation are described in references [3,4,5], where Ref.[3] contains FORTRAN codes and Ref.[4] comes with MATLAB codes while Ref. [5] demonstrates Mathematica codes.

III. MODEL

A one-dimensional current sheet model having 32 elements was employed and the magnetic field distribution near the surface was calculated at 32 locations to simulate measurement data (Fig. 1). The magnetic field distribution was given on a line that is parallel to the cross section of the current sheet. For simplicity, each measurement position is placed right above each current element. The magnetic fields and the system matrix A can be calculated by Ampere's law, assuming that the current flows at the middle of each element.

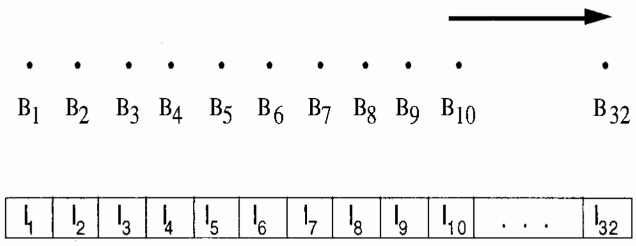

Fig. 1 One-dimensional current sheet model having 32 unknown currents. Tangential components of the magnetic fields are measured at 32 locations.

IV. NUMERICAL RESULTS AND DISCUSSIONS

When the distance between the current sheet and the measured line was less than a half width of each element of the current sheet, the system matrix A became singular. This is because the neighboring rows or columns of the system matrix turn to be almost identical. Fig. 2 shows an example of the 32 x 32 system matrix when the above distance equals the half width of the current elements.

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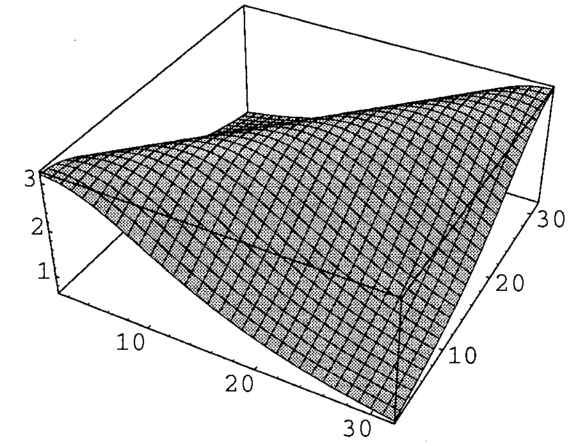
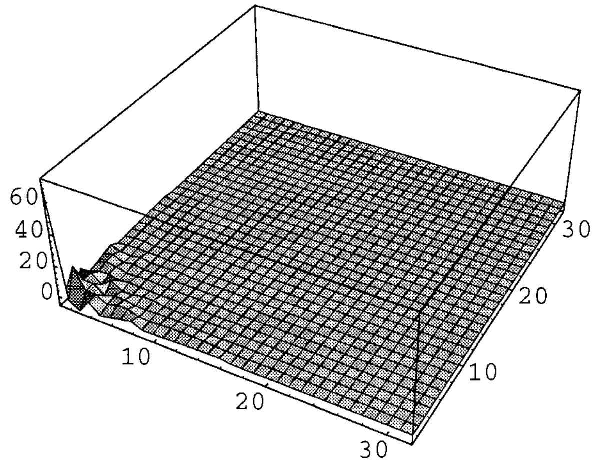


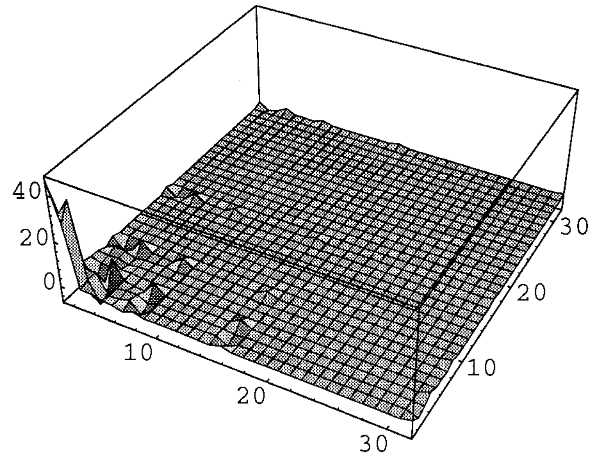
Fig. 2 The system matrix A of the model shown in Fig. 1

The wavelet transform approach is used to overcome this problem.

A. Comparison between Haar and Daubechies-4 wavelets



(a) Haar wavelets

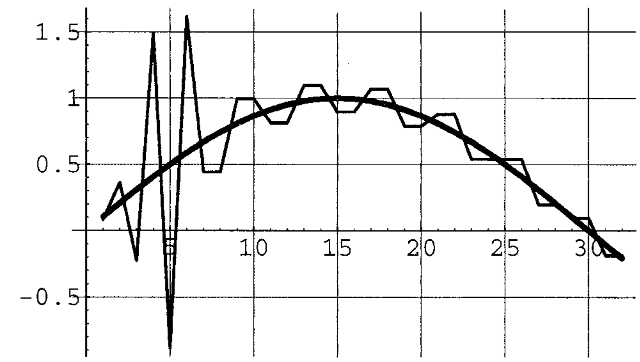


(b) Daubechies-4 wavelets

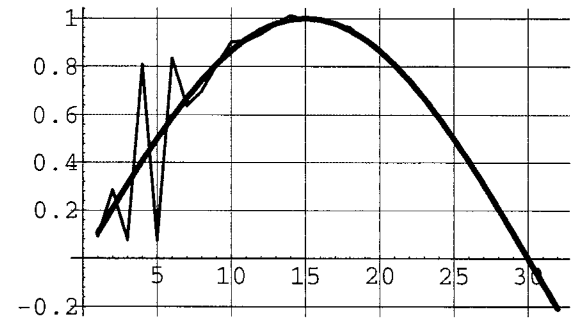
Fig. 3 Two dimensional wavelet transform of the system matrix A using (a) Haar and (b) Daubechies-4 wavelets

Fig. 3 (a) and 3(b) show two dimensional Haar and Daubechies-4 wavelet transforms of the system matrix A, each of which is A' whose dimension is 32 x 32. Taking only 19 x 19 elements from the left bottom corner of Fig. 3(a) resulted in a well-conditioned matrix, which is A" according to our notation. The condition of the system matrix was determined by calculating the relative error of the solution of the matrix equation [6]. Our criterion is to select the maximum matrix size when the relative error becomes smaller than 0.1%. In fact, this is the point that Mathemartica stops showing warning messages. Regarding Fig.3 (b), the above submatrix size was 18 x 18.

Fig. 4 shows the given (bold) and estimated (thin) current density distribution using Haar wavelets (a) and Daubechies-4 wavelets (b). The stair-like vibration in (a) results from a theoretical disadvantage of the Haar wavelets because we added zero components to construct the x' vector. Meanwhile, the Daubechies-4 wavelets provide more accurate results except for spike noises in the first half of the estimated current. This is another systematic noise due to wrap-around problems that is known in convolution calculation using FFT [3].



(a) Haar wavelets



(b) Daubechies-4 wavelets

Fig. 4 Estimated current distribution (thin line) using (a) Haar wavelets and (b) Daubechies-4 wavelets. Each bold line indicates the given current distribution.

The spike noise always appears in the first half of the solution vector, and thus we simply use the last half of the solution and repeat the same calculation after inverting the order of the measured magnetic field data as well as the system matrix. Then we delete the first half of the two solutions and combine the last half of the two solutions into one by re-inverting the order of the pre-inverted solution, leading to the spike-free result shown in Fig. 5.

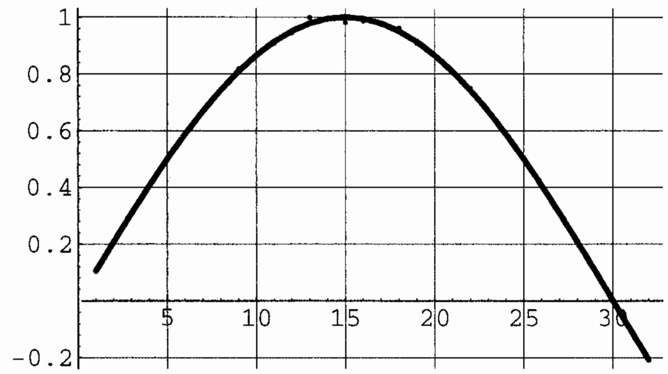


Fig. 5 Spike-free estimation by combining two solutions

B. Other spike-free procedures

We have discovered two other methods to remove the undesired spike noise. One method is to provide the field vector elements that fades out toward zero. A result using this technique is shown in Fig. 6. In the case of the model depicted in Fig. 1, this can be performed by placing the last a few measurement points far away from the current flowing conductor.

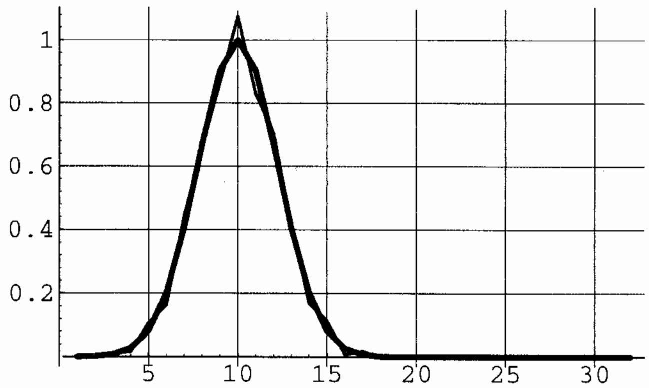


Fig. 6 A result using another spike-free procedure, where the field vector was constructed with the elements fading out toward zero. The bold line indicates the given field, while the thin line depicts the estimated result.

The other method is to make the sizes of the matrix and the solution vector less than power of two; subsequently, to add zeros to increase the sizes to power of two. A result using this technique is shown in Fig. 7.

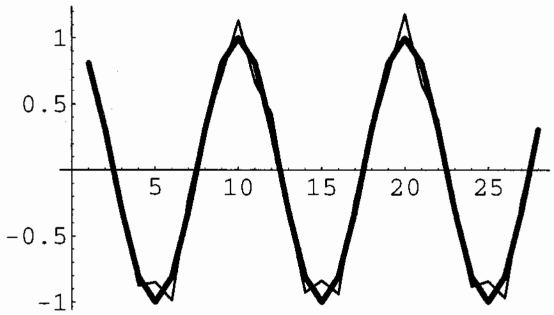


Fig. 7 A result using still another spike-free procedure, where the number of unknowns is less than power of two. The bold line indicates the given field, while the thin line depicts the estimated result.

The last two techniques essentially avoid the wrap-around problems by employing "zero-padding" [3]. This problem always happens when we use a cyclic matrix operator and an operated vector that ends with non-zero components. Lastly, all the computation has been done using *Mathematica* [7].

V.CONCLUSIONS

Inverse Problems of Vandermonde type systems have been solved using the discrete wavelet transform. Results were compared between two different wavelet basis functions, indicating that Daubechies-4 wavelets lead to much more accurate solutions than Haar wavelets. Three simple techniques for eliminating systematic noises are also proposed to further improve the accuracy of the final solution.

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