An application of the wavelets to the magnetic field source searching

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Recently, the wavelet analysis is being applied to the various fields, such as image data compression in informatics and spectrum analysis of the electrocardiogram. In the present article, we propose the two approaches, employing the wavelet analysis for the human heart diagnosis. One is the data base approach, and the other is an inverse approach searching for the magnetic field source of the human heart. The data base approach is an application of the data compression to the magnetocardiogram. Also, the magnetic field source searching is an application of the spectrum analysis to the magnetocardiogram. The results reveal that the data base approach makes it possible to identify the normal or abnormal heart, and the magnetic field source search is capable of estimating the current distribution of a distinct heart. [S0021-8979(96)79408-1]

I. INTRODUCTION

Recent investigations concerning the magnetocardiogram (MCG) are establishing their possible use for human heart diagnosis, because the MCG data contain the activation current information of the human heart.[1] In order to utilize the MCG data for heart diagnosis, it may be classified by two approaches; one is a direct reading of the information from the MCG data, and the other is an inverse approach searching for the activation current in the heart. Direct application of the MCG data for diagnosis requires a great deal of experience and skillfulness for reading the information included in the MCG data. On the other hand, the inverse approach does not require intensive experience, but it is essential to exploiting a solution strategy for the inverse source problem.[2–4]

Recently, the wavelet analysis has been developed for the image data compression in informatics and for the spectrum analysis of the ECG.[5,6]

In the present article, we applied the wavelet analysis to the human heart diagnosis. The wavelet analysis is applied to the MCG data compression for the data base oriented diagnosis. This proposes an expert system employing the wavelet analysis. Further, the wavelet analysis is applied to the inverse approach searching for the current distribution in the heart.

As for results, the data base approach makes it possible to identify the normal or abnormal heart. Also, it is shown that the magnetic field source searching approach is capable of estimating the current distribution of the distinct heart.

Thus, we show that our approaches employing the wavelets are quite effective tools for MCG diagnosis.

II. APPLICATION OF THE WAVELET ANALYSIS TO MCG DIAGNOSIS

A. Discrete wavelet transform

In the present article, we employ Daubechie's analyzing wavelets. Let us consider the following linear transformation:

$$X' = CX, \tag{1}$$

where $\mathbf{X}$ is a data vector with order of n; n must be a power of 2; and C is

$$C = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 & 0 & 0 & \cdot & 0 & 0 & 0 & 0 \\ c_3 & -c_2 & c_1 & -c_0 & 0 & 0 & \cdot & 0 & 0 & 0 & 0 \\ 0 & 0 & c_0 & c_1 & c_2 & c_3 & \cdot & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & -c_2 & c_1 & -c_0 & \cdot & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdot & c_0 & c_1 & c_2 & c_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdot & c_3 & -c_2 & c_1 & -c_0 \\ c_2 & c_3 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & c_0 & c_1 \\ c_1 & -c_0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 & c_3 & -c_2 \end{bmatrix}. \tag{2}$$

In Eq. (2), the first, third, fifth, and the other odd rows generate the components of data convolved with the coefficients c_0, c_1, c_2, c_3. This corresponds to a weighted integral operation. On the other side, the even rows generate the com-

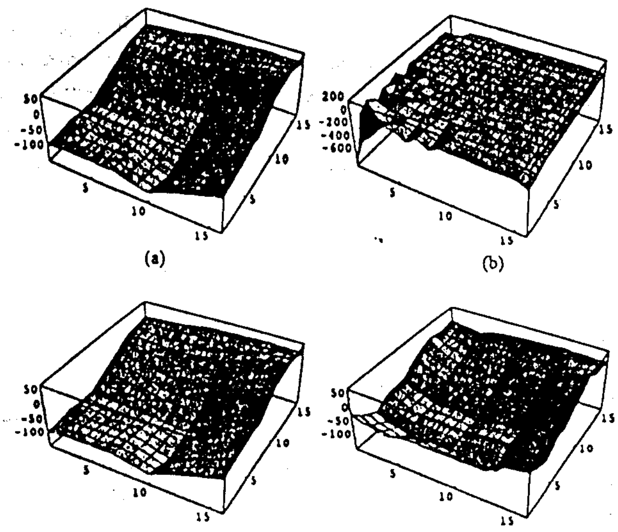


FIG. 1. An example of MCG data compression by means of the wavelet transformation. (a) An original MCG map, (b) a wavelet spectrum of (a), (c) a reproduced MCG map from the 100% spectrum in (b), and (d) the reproduced MCG map from the 6.25% spectrum in (b).

ients of data convolved with the coefficients $c_3, -c_2, c_1, -c_0$. This corresponds to a weighted differential operation.

In order to carry out an inverse linear transformation, the coefficients c_0, c_1, c_2, c_3 should be determined by

$$C^T C = I, \tag{3}$$

where I is a nth order unit matrix and the superscript T refers to the transpose of matrix C^T in Eq. (3):

$$C^T = \begin{bmatrix} c_0 & c_3 & 0 & 0 & \cdot & 0 & 0 & 0 & 0 & c_2 & c_1 \\ c_1 & -c_2 & 0 & 0 & \cdot & 0 & 0 & 0 & 0 & c_3 & -c_0 \\ c_2 & c_1 & c_0 & c_3 & \cdot & 0 & 0 & 0 & 0 & 0 & 0 \\ c_3 & -c_0 & c_1 & -c_2 & \cdot & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & c_2 & c_1 & c_0 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdot & c_3 & -c_0 & c_1 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdot & 0 & 0 & c_2 & c_1 & c_0 & c_3 \\ 0 & 0 & 0 & 0 & \cdot & 0 & 0 & c_3 & -c_0 & c_1 & -c_2 \end{bmatrix} \tag{4}$$

From Eqs. (2)–(4), we have

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = 1, \qquad c_2 c_0 + c_3 c_1 = 0. \tag{5}$$

Equation (5) is composed of the two equations and has the four unknowns c_0, c_1, c_2, c_3. To determine the coefficients c_0, c_1, c_2, c_3, we have to consider the following conditions:

$$c_3 - c_2 + c_1 - c_0 = 0, \qquad 0c_3 - 1c_2 + 2c_1 - 3c_0 = 0. \tag{6}$$

From Eqs. (5) and (6), we have

$$c_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, \quad c_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}, \quad c_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}, \quad c_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}. \tag{7}$$

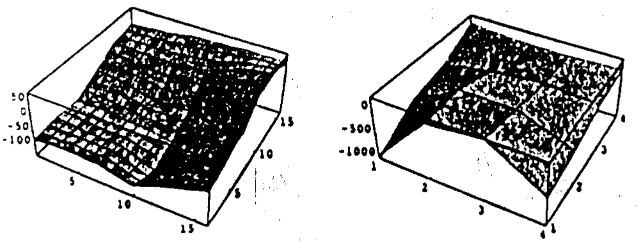
The set of coefficients c_0, c_1, c_2, c_3 in Eq. (7) is Daubechie's fourth-order analyzing wavelets.

For simplicity, let us consider a data vector X with the order of 16:

$$\mathrm{X} = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ x_{11}\ x_{12}\ x_{13}\ x_{14}\ x_{15}\ x_{16}]^T. \tag{8}$$

In the case, the linear transformation using the matrix C_{16} yields

$$\mathrm{X}' = C_{16}\mathrm{X} = [s_1\ d_1\ s_2\ d_2\ s_3\ d_3\ s_4\ d_4\ s_5\ d_5\ s_6\ d_6\ s_7\ d_7\ s_8\ d_8]^T. \tag{9}$$



(a) Original MCG

(b) Compressed MCG in the form of wavelet spectrum

FIG. 2. The data compression of the MCG map of normal heart at QRS 30.0 (ms). (a) An original MCG map, and (b) the compressed MCG map in the form of wavelet spectrum.

The elements in vector X′ are sorted by using the following matrix:

$$P_{16} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{10}$$

Thus, we have

$$P_{16}\mathrm{X} = P_{16}C_{16}\mathrm{X}$$
$$= [s_1\ s_2\ s_3\ s_4\ s_5\ s_6\ s_7\ s_8\ d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7\ d_8]^T. \tag{11}$$

Further transformation to the elements $s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8$ in Eq. (11) yields

$$W^{(2)}\mathrm{X} = [S_1\ S_2\ S_3\ S_4\ D_1\ D_2\ D_3\ D_4\ d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7\ d_8]^T. \tag{12}$$

Similar transformation to s_1, s_2, s_3, s_4 in Eq. (12) yields

$$W^{(3)}\mathrm{X} = [\mathrm{S}_1\ \mathrm{S}_2\ \mathrm{D}_1\ \mathrm{D}_2\ D_1\ D_2\ D_3\ D_4\ d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7\ d_8]^T. \tag{13}$$

The transformation matrices used in Eqs. (12) and (13) are

$$W^{(2)} = (P'_{16}C'_{16})(P_{16}C_{16}), \quad W^{(3)} = (P''_{16}C''_{16})(P'_{16}C'_{16})(P_{16}C_{16}), \tag{14}$$

$$P'_{16} = \begin{bmatrix} P_8 & 0 \\ 0 & I_8 \end{bmatrix}, \quad C'_{16} = \begin{bmatrix} C_8 & 0 \\ 0 & I_8 \end{bmatrix},$$

$$P''_{16} = \begin{bmatrix} P_4 & 0 \\ 0 & I_{12} \end{bmatrix}, \quad C''_{16} = \begin{bmatrix} C_4 & 0 \\ 0 & I_{12} \end{bmatrix}. \tag{15}$$

Equation (13) is the finally obtained wavelet spectrum. The elements $\mathrm{S}_1\mathrm{S}_2$ in (13) are called the Mother Wavelet coefficients, and the others are called the wavelet coefficients at each level.

Inverse wavelet transform is carried out by

$$\mathrm{X} = [W^{(3)}]^T W^{(3)}\mathrm{X},$$
$$[W^{(3)}]^T = [(P''_{16}C''_{16})(P'_{16}C'_{16})(P_{16}C_{16})]^T,$$
$$= (P_{16}C_{16})^T (P'_{16}C'_{16})^T (P''_{16}C''_{16})^T,$$
$$= C_{16}^T P_{16}^T (C'_{16})^T (P'_{16})^T (C''_{16})^T (P''_{16})^T. \tag{16}$$

B. MCG data compression

Figure 1 shows an example of MCG data compression by means of the wavelet transformation. Figure 1(a) shows the original MCG map of the normal heart at QRS 30.0 (ms). Figure 1(b) shows a wavelet spectrum of Fig. 1(a). Figures

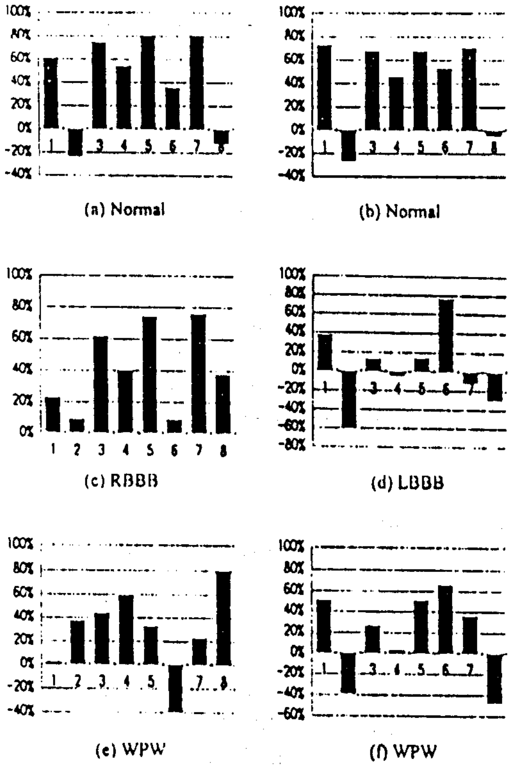


FIG. 3. Examples of the MCG diagnosis by means of the data base approach. (a) The correlative coefficient between the data base and normal heart. Similarly the correlative coefficients to the other normal (b), RBBB (c), LBBB (d), WPW syndromes (e), and the other WPW syndromes (f).

1(c) and 1(d) show the reproduced MCG map from the 100% and 6.25% spectrums in Fig. 1(b), respectively.

The results in Fig. 1 show that the wavelet transformation efficiently compresses the information included in the MCG map.

C. Data base approach

We applied the data compression by the wavelet transformation to the practical MCG maps.

Figures 2(a) and 2(b) show the original MCG map and its compressed wavelet spectrum (6.25%), respectively. We constructed the compressed data base using eight sets of the MCG data of normal hearts.

Figure 3(a) shows the correlative coefficients between the data base and the normal heart. Figures 3(b)–3(f) show the correlative coefficients to the other normal heart, right bundle branch block (RBBB), left bundle branch block (LBBB), Wolff–Perkinson–White (WPW) syndromes, and the other WPW syndromes, respectively.

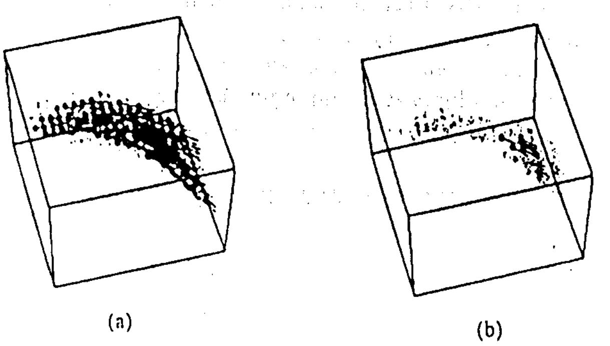


FIG. 4. Estimated current distribution in the case of normal heart at QRS 30.0 (ms). (a) An original current distribution, and (b) reproduced current distribution from the 1/16 times compressed data base.

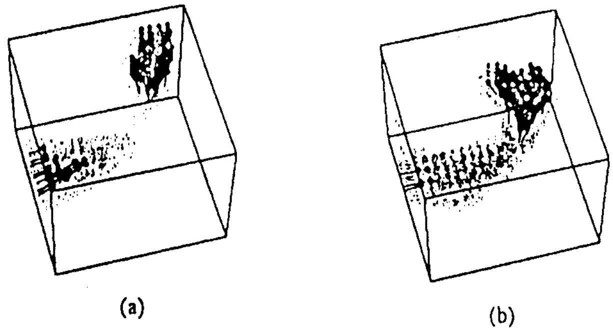


FIG. 5. Estimated current distribution in the case of WPW syndromes at QRS 30.0 (ms). (a) An original current distribution, and (b) reproduced current distribution from the 1/16 times compressed data base.

Consideration of the results in Fig. 3 suggests that the data base approach makes it possible to identify the normal or not normal heart. Further, increasing a number of data bases may improve the accuracy of this approach.

D. Magnetic field source searching approach

In order to improve the data base approach, we evaluated the current distribution of heart from the compressed data base. Our searching strategy of the current distribution is called the sampled pattern matching method, which is a kind of generalized factor analysis.[2–4]

Figures 4(a) and 4(b) show an original current distribution and its reproduced current distribution from the 6.25% compressed data base, respectively. A similar example is shown in Fig. 5 for the WPW syndromes. Considering the results in Figs. 4 and 5, it is confirmed that the current distribution in the heart can be reproduced with sufficient accuracy from only the 1/16 times compressed data base. This means that further improvement of the heart diagnosis can be carried out by the inverse approach.

III. CONCLUSION

In the present article, we have applied the wavelet analysis to the MCG diagnosis.

At first, we proposed the data base approach using the MCG data compression by wavelet transformation. Using this data base approach, the normal and abnormal hearts could be read off.

Second, applying the magnetic field source searching approach to the compressed data base, the distinct current distribution reveals the details of heart condition. A further increase of the number of data bases should be carried out for the improvement of accurate diagnosis.

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