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Generalized Vector Sampled Pattern Matching Method –Theory and Applications

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Abstract. A powerful iterative solver for system of equations has been developed. This solver, Generalized Vector Sampled Pattern Matching (GVSPM), enables us to obtain the solution of any ill-posed linear system equations, e.g., having rectangular or singular matrix. The key idea is that the objective function is the angle obtained by an inner product between the input vector and solution system of equations. This paper reviews the Generalized Sampled Pattern Matching (GSPM) method, which is the original of proposed method, and then introduces the GVSPM method along with solution algorithm. The theoretical background and applications to inverse source problems on magnetic fields are described. From the applications, the GSPM and GVSPM methods tend to lead the discontinuous and continuous solutions, respectively. However, the GVSPM has the advantages of stability and convergence.

1. Introduction

When we address the non-destructive evaluation (NDE) and electromagnetic compatibility (EMC) problems, it is essential to reduce into solving for inverse problems. Because of the nature of their formulations, the number of equations and that of solutions are not always the same. Therefore, grappling with the inverse problems results in solving for ill-posed linear system of equations. In order to solve them, some approaches may be considered. For instance, least square approach is applicable when the number of equations is greater than that of solutions. Inversely, when the number of solutions is greater than that of equations, some constraint conditions should be imposed [1]. In addition, the solution by the neural network (NN) has dependence on the number of training as well as structure of the networks.

The early 1990's, Saito proposed a novel solution strategy called Sampled Pattern Matching (SPM) method to apply the inverse problems which result in solving for the ill-posed linear system of equations. The principle is that the patterns of column vectors constituting the system matrix are investigated. This idea has worked out the electric as well as magnetic source searching problems [2][3], shape design of magnetic cores [4] and so on. Afterward, this inverse problem solver was generalized to solve for various types of linear system of equations. The generalized solver, Generalized Sampled Pattern Matching (GSPM), has been applied to optimize the dose of electron beams [5][6] and so on. Though the GSPM is possible to solve for any types of linear equations, the convergence process as well as mathematical background was unclear at the time. Recently, mathematical background has been clarified, and the convergence process has been proved [7]. This leads to improvement of the GSPM.

This paper reviews the GSPM method, and then introduces the improved method, Generalized Vector Sampled Pattern Matching (GVSPM). The key idea of the GVSPM is that the objective function is the angle obtained by means of inner product between the input vector and solution system of equations. The convergence process is described in analytical way. The applications and comparisons of the GSPM and GVSPM are demonstrated along with the inverse source problems on magnetic fields. As a result, the applications reveal that the GSPM and GVSPM methods tend to lead the discontinuous and continuous solutions, respectively. Moreover, the solution by means of GVSPM converges with smaller number of iterations for computation.

2. Generalized Sampled Pattern Matching (GSPM) Method

2.1 Basic Equation

Solving the inverse problems results in handling the ill-posed linear system of equations. The basic equation we have to solve is as follows:

$$\mathbf{Y} = C\mathbf{X}, \tag{1}$$

where **Y** and **X** denote the n-th order input- and m-th order solution/output- vectors, respectively; C is an n by m rectangular matrix. Even if $m=n$ case, the matrix C is not always positive definite due to the formulation of inverse problems. (1) can be rewritten by

$$\mathbf{Y} = \sum_{i=1}^{m} x_i \mathbf{C}_i, \quad \mathbf{X} = [x_1 \quad x_2 \quad . \quad x_m]^T, C = [\mathbf{C}_1 \quad \mathbf{C}_2 \quad . \quad \mathbf{C}_m]. \tag{2}$$

(2) means that the input vector **Y** is represented by means of linear combination of column vectors $\mathbf{C}_i$, i=1, 2,..., m, in the system matrix C. The principle of the SPM method is to search for the patterns representing by the column vectors $\mathbf{C}_i$s in order to satisfy the input vector **Y** in (1).

2.2 GSPM Method

In the GSPM, the elements in solution vector **X** are approximated by quantized discrete values as step width. At first, matching rates between the vector **Y** and column vector $\mathbf{C}_i$, i=1, 2,..., m, are calculated using the Cauchy-Schwarz formula given as (3) to select the most dominant pattern.

$$\gamma_i = \frac{\mathbf{Y} \bullet \mathbf{C}_i}{|\mathbf{Y}||\mathbf{C}_i|}, \quad i = 1,2,...,m, \tag{3}$$

where γ is called pattern matching index. If a column vector $\mathbf{C}_i$ is the most dominant, then correspoding element of the solution vector **X** is increased by the quantized discrete value. From the next calculation steps, the searching patterns are overlapped by the selected column vector, namely,

$$\gamma_i^{(k)} = \frac{\mathbf{Y} \bullet [\mathbf{C}_i + \mathbf{C}_k]}{|\mathbf{Y}||\mathbf{C}_i + \mathbf{C}_k|}, \quad i = 1,2,...,m, \tag{4}$$

where the superscript (k) refers to the k-th iteration. Moreover, $\mathbf{C}_k$ is the selected pattern in terms of k-1-th iteration. (4) means that the amplitude of the solution is approximated by the concentrating rate of similar solution pattern of $\mathbf{C}_k$. The calculation continues until the pattern matching index $\gamma_i^{(k)}$ takes 1. This means the obtained pattern $(\mathbf{C}_i+\mathbf{C}_k)$ gives the best pattern matching to the input vector **Y**. Finally, increment the elements constituting the output vector **X** in each of iterations yields the GSPM solution. In this method, calculating (4) carries out to each of column vectors, therefore, it is essentially required enormous time and number of iterations for computation.

3. Generalized Vector Sampled Pattern Matching (GVSPM) Method

3.1 Key Idea

Normalzing (2) by the input vector amplitude $|\mathbf{Y}|$ gives the following relationship:

$$\frac{\mathbf{Y}}{|\mathbf{Y}|}=\sum_{i=1}^{m} x_i \frac{|\mathbf{C}_i|}{|\mathbf{Y}|}\frac{\mathbf{C}_i}{|\mathbf{C}_i|} \quad \text{or} \quad \mathbf{Y}'=C'\mathbf{X}', \tag{5}$$

where the prime (') denotes the normalized quantities. (5) means that the normalized input vector **Y'** is obtained as a linear combination of the weighted solutions $x_i|\mathbf{C}_i|/|\mathbf{Y}|$, i=1, 2,..., m, with the normalized column vectors $\mathbf{C}_i/|\mathbf{C}_i|$, i= 1, 2,..., m. It should be noted that the solution vector **X** could be obtained when an inner product between **Y'** and $C'\mathbf{X}'$ becomes 1. This is the key idea of the GVSPM method.

3.2 Objective Function

Define a function f derived from an angle between the input vector **Y** and $C\mathbf{X}^{(k)}$ given in terms of the k-th iterative solution $\mathbf{X}^{(k)}$, as given by

$$f\left(\mathbf{X}^{(k)}\right)=\frac{\mathbf{Y}}{|\mathbf{Y}|}\bullet\frac{C\mathbf{X}^{(k)}}{\left|C\mathbf{X}^{(k)}\right|}=\mathbf{Y}'\bullet\frac{C'\mathbf{X}'^{(k)}}{\left|C'\mathbf{X}'^{(k)}\right|}. \tag{6}$$

Then the solution $\mathbf{X}^{(k)}$ is obtained when the function $f(\mathbf{X}^{(k)})$ converges to

$$f\left(\mathbf{X}^{(k)}\right)\rightarrow 1. \tag{7}$$

This is the objective function of the GVSPM solution. The objective function of the GSPM is the angle between the input vector and pattern incrementally composed of column vectors while those of the GVSPM is the angle between the normalized input vector and output system of equations. Thereby, evaluation of (7) needs only once an iteration.

3.3 Iteration Algorithm

Let $\mathbf{X}'^{(0)}$ be an initial solution vector given by

$$\mathbf{X}'^{(0)}=C'^{T}\,\mathbf{Y}', \tag{8}$$

then the first deviation vector $\Delta\mathbf{Y}'^{(1)}$ is obtained as

$$\Delta\mathbf{Y}'^{(1)}=\mathbf{Y}'-\frac{C'\mathbf{X}'^{(0)}}{\left|C'\mathbf{X}'^{(0)}\right|}. \tag{9}$$

When the deviation $\Delta\mathbf{Y}'$ becomes zero vector, the objective function (7) is automatically satisfied. Modification by the deviation vector $\Delta\mathbf{Y}'^{(k-1)}$ gives the k-th iterative solution vector $\mathbf{X}'^{(k)}$, namely,

$$\begin{aligned}\mathbf{X}'^{(k)} &= \mathbf{X}'^{(k-1)} + C'^{T}\,\Delta\mathbf{Y}'^{(k-1)} \\ &= C'^{T}\,\mathbf{Y}' + \left(I_m - \frac{C'^{T} C'}{\left| C'\mathbf{X}'^{(k-1)} \right|} \right)\mathbf{X}'^{(k-1)},\end{aligned} \tag{10}$$

where I_m denotes a m by m unit matrix.

3.4 Convergence Condition

The convergence condition of the GVSPM iterative strategy is that the modulus of all characteristic values of state transition matrix in (10) must be less than 1 [8]. The state transition matrix S is given by

$$S = I_m - \frac{C'^{T} C'}{\left| C'\mathbf{X}'^{(k-1)} \right|} = I_m - \frac{C'^{T} C'}{\left| \mathbf{Y}'^{(k-1)} \right|}. \tag{11}$$

Since the vector $\mathbf{Y}'^{(k-1)}$ is normalized, (11) can be rewritten by

$$S = I_m - C'^{T} C'. \tag{12}$$

This means the convergence condition is independent of initial solution in (8). Furthermore, the solution is always evaluated by means of (6) so that the solution only depends on the system matrix C.

Let λ be characteristic value of the state transition matrix S. Then the determinant of symmetrical matrix is obtained:

$$\left| \lambda I_m - S \right| = \begin{vmatrix} \lambda & \varepsilon_{12} & \cdot & \varepsilon_{1m} \\ \varepsilon_{12} & \lambda & \cdot & \varepsilon_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ \varepsilon_{1m} & \varepsilon_{2m} & \cdot & \lambda \end{vmatrix} = 0. \tag{13}$$

It is obvious that the moduli of off-diagonal elements in (13) take less than 1 because of the normalized column vectors of matrix C', namely,

$$\left| \varepsilon_{ij} \right| < 1, \qquad i=1,2,\ldots,m,\ j=1,2,\ldots,m. \tag{14}$$

Suppose the modulus characteristic value $|\lambda|$ takes more than 1. Then the column vectors in (13) become linear independence because of (14). In such a case, the determinant in (13) is not zero so that the condition $|\lambda| < 1$ should be satisfied. Therefore, it is proved that the GVSPM is always carried out on stable iteration.

4. Applications

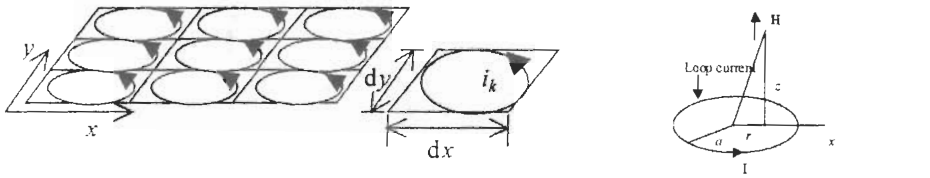
4.1 Visualization of 2-Dimensional Current Distribution

To visualize the current vector distribution from locally measured magnetic field is one of the typical inverse problems and fundamental of NDE. Let us consider a unit loop current model illustrated in Fig. 1[9]. The magnetic field H caused by a loop coil shown in Fig. 1(b) can be analytically obtained as follows:

$$H = \frac{I}{2\pi}\left[\frac{1}{\sqrt{(a+r)^2+z^2}}\right]\left[\frac{a^2-r^2-z^2}{(a-r)^2+z^2}E(\kappa)+K(\kappa)\right]$$

$$\kappa^2 = \frac{4ra}{(r+a)^2+z^2}, \qquad (15)$$

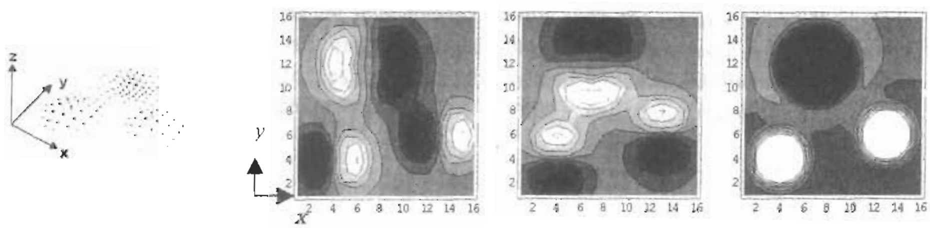
where $K(\kappa)$ and $E(\kappa)$ are the first- and second- kind elliptic integrals, respectively. The system of equations is based on (15) in this example. In this case, the vectors **Y** and **X** in (1) are composed of the measured magnetic field H and current I expressed by (15), respectively.



(a) Unit loop array (b) An element of the unit loop array

Fig. 1 Unit loop current model

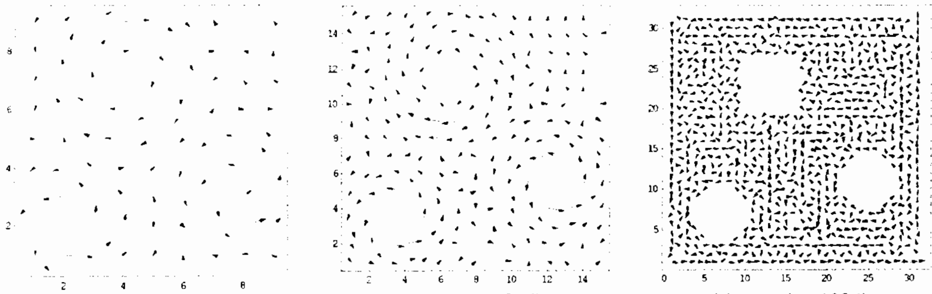
Fig. 2 shows the measured magnetic field distribution generated by 3 excited coils. At first, each component of the 3-dimensional magnetic field is measured at a surface over the target, as shown in Figs. 2(b)-(d). Number of measured points is 16 by 16. Therefore, the input vector **Y** in (1) becomes 256th order, namely n=256. Second, the system of equations obtained by (15) is solved in terms of the z component of magnetic field by the GSPM and GVSPM methods. The estimation points are set to 10 by10, 16 by 16 and 32 by 32 in this example. In these cases, numbers of elements in output vectors are 100, 256 and 1024. Finally, 2-dimensional current vector distribution can be obtained as in Figs. 3 and 4.



(a) Vector representation (b) x component (c) y component (d) z component

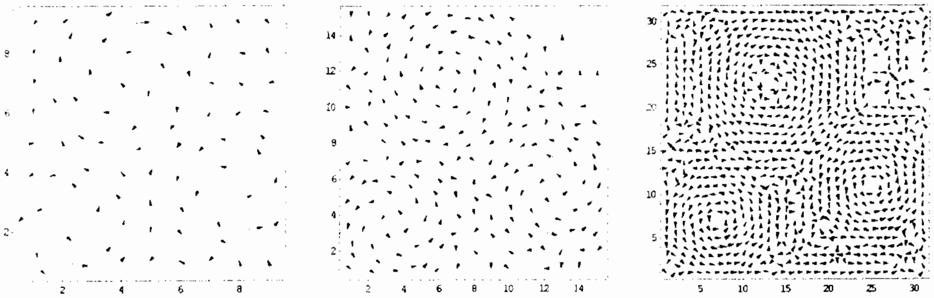
Fig. 2 Measured magnetic field of 3 excited coils

The GSPM and GVSPM realize Figs. 3 and 4, respectively. In cases of $m<n$ and $m=n$, each of the solutions yields fairly good approximation. On the other hand, in case of $m>n$, the GVSPM gives better result than the GSPM in respect of current flowing directions. Since the GSPM method carries out element-to-element evaluation described as (4), then it needs a large number of iterations. The results of GSPM and GVSPM have spent 200000 and 100 iterations on computation, respectively. The GVSPM realizes faster computation as well as good approximation in this case.



(a) $m<n$ (m=100) (b) $m=n$ (m=256) (c) $m>n$ (m=1024)

Fig. 3 2-dimensional current vector estimation by means of GSPM (n=256)



(a) $m<n$ (m=100) (b) $m=n$ (m=256) (c) $m>n$ (m=1024)

Fig. 4 2-dimensional current vector estimation by means of GVSPM (n=256)

4.2 Current Position Searching by Deconvolution

Next, searching the current flowing position is carried out using measured magnetic field. Information coming from the current can only be observed by specific sensor such as solenoidal coils. In most cases, the current distribution is observed by sensor signal including some physical space characteristics such as Green's relationship.

Let us consider the one dimensional magnetic field distribution scanned by a solenoidal sensor coil. Then, the output voltage would be obtained like Fig. 5, and it takes maximum voltage at the position 100 cm. Using this physical space characteristic of the sensor coil, the exact current flowing positions are investigated. In this case, the vectors **Y** and **X** in (1) are corresponding to the measured magnetic field obtained by the sensor and the exact current distribution to be searched, respectively. The system matrix C, shown in Fig. 6, is composed of the physical space characteristic. This scheme is called deconvolution.

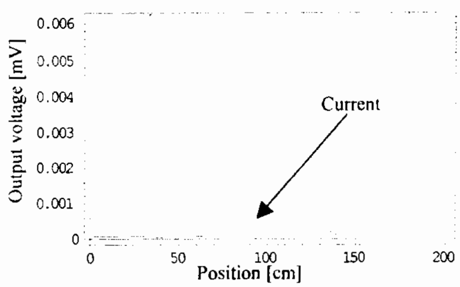


Fig. 5 Physical space characteristic of a sensor coil

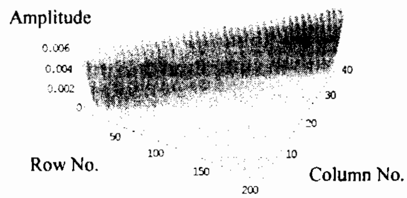


Fig. 6 Elements of system matrix

Fig. 7 shows the measured magnetic field caused by two current lines. The measurement is carried out by a solenoidal coil over the current line surface. Number of measured points is 41 and number of measured points of physical space characteristic is 201. Therefore, **Y** and **X** respectively become the vectors with 41 and 201 orders, namely, m=201, n=41. By means of the GSPM and GVSPM, Fig. 8 shows the deconvoluted results. The result of GSPM in Fig. 8(a) reveals the positions of the exact currents has been estimated with much higher accuracy than that of GVSPM in Fig. 8(b). This nature is caused by iteration algorithm. Because the GSPM method carries out element-to-element evaluation, the solution having discontinuous characteristics is easy to be obtained. On the other hand, since the GVSPM evaluates with vector type iteration described above, then it tends to be continuous solutions. The results of GSPM and GVSPM have spent 200000 and 100 iterations on computation, respectively.

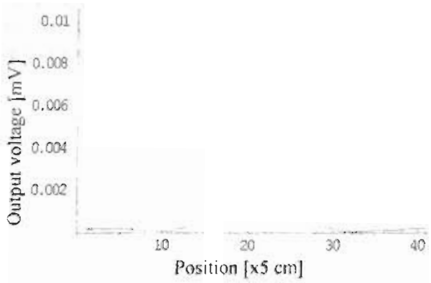
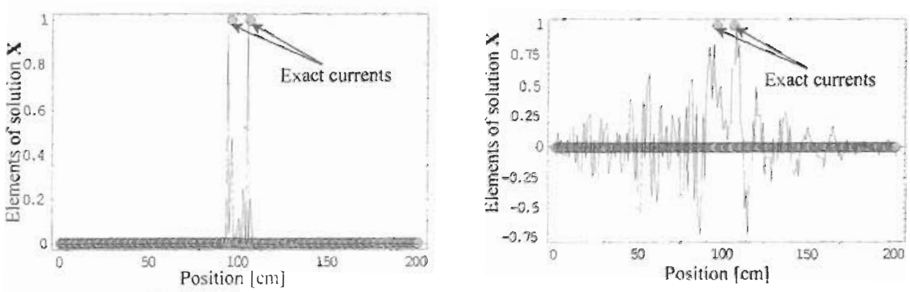


Fig. 7 Measured magnetic field distribution by the sensor coil



(a) GSPM method (b) GVSPM method

Fig. 8 Current flowing position searching by means of deconvolution

5. Conclusions

This paper has introduced the GVSPM as one of the powerful iterative solvers for linear system of equations. The original version of the GSPM requires an enormous computation time while the GVSPM algorithm makes it possible to reduce the computation time and the reliablity of the solution is clearly verified. The distinguished feature is that the objective function is the angle between the right and left hand terms in linear system of equations. This makes it possible to solve any types of simultaneous equations, i.e., having row- as well as column- wide type rectangular system matrices. The convergence process has been described analytically. As the applications of GSPM and GVSPM, the current searching problems from locally measured magnetic fields have demonstrated the usefulness of our solution strategy. Solving for the ill-posed system of equations is inevitable for the inverse problems. However, the GSPM and GVSPM make it possible to select the physically existing solution although this solver never uses the matrix inversion. Particularly, the GSPM and GVSPM tend to be discontinuous and continuous solution, respectively.

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