Dynamics on Ferroresonant Circuit Exhibiting Chaotic Phenomenon

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***Abstract*—This paper studies dynamics of a series ferroresonant circuit. A Chua type magnetization model derives a state variable equation with nonlinear parameters. Backward Euler method with adaptive stepsize control performs a transient analysis. Modal analysis to the state variable equation is also carried out in each calculation period of the Euler method in order to obtain the condition of internal system. Finally, it is clarified that the characteristic values vary on the regular loci although the chaotic phenomenon is exhibiting. The relation between the time constants and operating frequencies is shown to clarify the ferroresonant mode.**

***Index Terms*—Chaotic phenomenon, Chua-type magnetization model, modal analysis, series ferroresonant circuit.**

I. Introduction

VERSATILE capability of the modern ferromagnetic materials has improved efficiency in various types of electrical apparatus. On the other hand, systems having ferromagnetic materials occasionally cause unpredictable behavior due to magnetic hysteresis, saturation, eddy current, and so on. Thus, the nonlinear calculations are essentially required for electrical apparatus design in order to avoid such unpredictable behavior.

The present paper studies dynamics of a series ferroresonant circuit as shown in Fig. 1, which is available as constant current power supplies. To clarify the dynamics of ferroresonant circuit precisely, we carry out a transient analysis fully taking into account hysteretic properties of magnetodynamics [1]–[4]. A Chua-type magnetization model is employed to derive the circuit equations as a state variable equation [5], [6]. To analyze the ferroresonant mechanism, the characteristic values of the state variable equation are calculated in each period of the backward Euler iteration. It reveals that the changes of characteristic values have no hysteretic properties even though a chaotic phenomenon is exhibiting. The response of ferroresonant system is divided into two typical modes by means of modal analysis. The relation between the time constants and operating frequencies is shown to clarify the ferroresonant mode.

II. Transient Analysis of Ferroresonant Circuit

A. Chua-Type Magnetization Model and Its Parameters

To carry out transient analysis of ferroresonant circuit, we employ a Chua-type magnetization model representing dynamic

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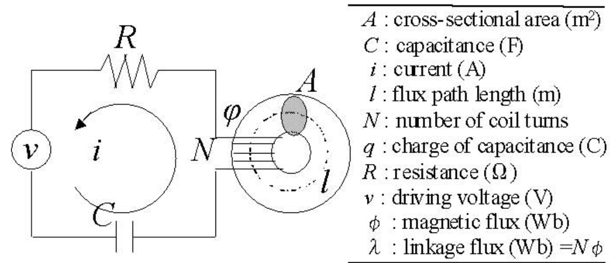


Fig. 1. Series ferroresonant circuit.

constitutive relation between magnetic field H (A/m) and flux density B(T)

$$H + \frac{\mu_r}{s}\frac{dH}{dt} = \frac{1}{\mu}B + \frac{1}{s}\frac{dB}{dt} \quad (1)$$

where μ, μ_r, and s denote permeability (H/m), reversible permeability (H/m), and hysteresis parameter (Ω/m), respectively [3], [5]. Fig. 2 shows the curves giving these parameters for soft iron used in this paper. A significant feature of these parameters is that they are determined independently to the past magnetization histories because of the ideal magnetization curve approach, as pointed out by Bozorth [7]. The validity of this model has been verified by the experiments of the typical magnetization characteristics excepting for anisotropic materials and permanent magnets [6], [8]–[10].

B. Formulation

Consider the series ferroresonant circuit shown in Fig. 1. At first, the line integral of (1) along with flux path l yields magnetic motive force. Thus, the relation between the current i and linkage flux λ of the inductor is given by

$$Ni + \frac{\mu_r}{s}N\frac{di}{dt} = \frac{l}{\mu AN}\lambda + \frac{l}{sAN}\frac{d\lambda}{dt}. \quad (2)$$

Moreover, the relation between the driving voltage source v and current i is derived from the circuit connection

$$i = \frac{1}{R}\left(v - \frac{1}{C}q - \frac{d\lambda}{dt}\right). \quad (3)$$

Second, substituting (2) into (3) yields the state equations

$$\frac{\mu_r}{s}\frac{d^2\lambda}{dt^2} + \left(1 + \frac{lR}{sAN^2} - \frac{\mu_r}{sRC}\right)\frac{d\lambda}{dt} + \frac{lR}{\mu AN^2}\lambda + \frac{1}{C}\left(1 - \frac{\mu_r}{sRC}\right)q = \frac{\mu_r}{s}\frac{dv}{dt} + \left(1 - \frac{\mu_r}{sRC}\right)v \quad (4)$$

$$\frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\lambda}{dt} + \frac{1}{C}q - v\right). \quad (5)$$

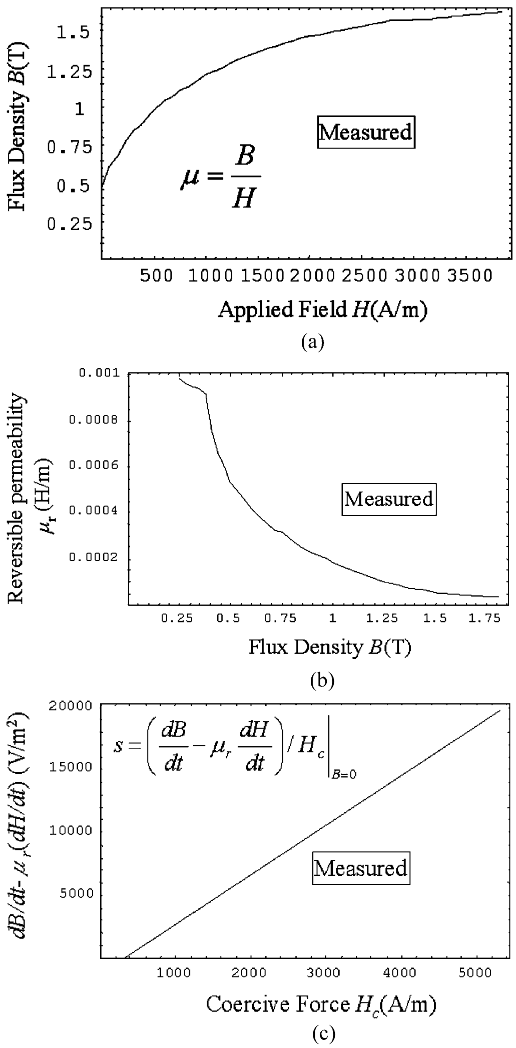


Fig. 2. Parameters of Chua-type magnetization model (Measured: soft iron). (a) Permeability μ. (b) Reversible permeability μ_r. (c) Hysteresis parameter s.

Finally, the state equations (4) and (5) yield a state variable equation having 3×3 square matrix a

$$\frac{d}{dt}\begin{pmatrix} \lambda \\ d\lambda/dt \\ q \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ a_{12} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \lambda \\ d\lambda/dt \\ q \end{pmatrix} + \begin{pmatrix} 0 \\ u_2 \\ u_3 \end{pmatrix} \tag{6}$$

or

$$\frac{d}{dt}\mathbf{x} = a\mathbf{x} + \mathbf{b} \tag{7}$$

where the elements $a_{21}, a_{22}, \ldots, u_2$ and u_3 are determined by (4) and (5).

C. Backward Euler Method With Adaptive Stepsize Control

Backward Euler strategy to numerical solution of (7) enables us to carry out transient analysis of the ferroresonant circuit [11]. As illustrated in Fig. 3, the calculation compares two solutions in each calculation period [12]. One is calculated with time step width Δt(s), and the other is with $\Delta t/2$. Evaluate the difference ε between these two solutions, then the solution of the period is determined. If the difference ε is greater than a criterion E

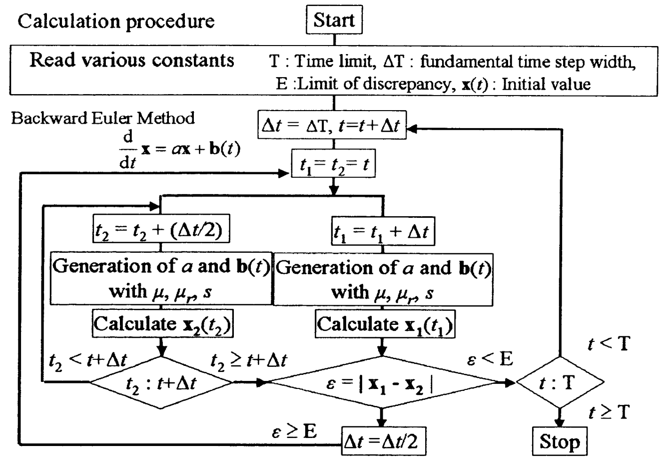


Fig. 3. Flowchart of the calculation with adaptive stepsize control.

TABLE I
CONSTANTS FOR ANALYSIS OF FERRORESONANT CIRCUIT

μ : permeability (H/m)	Fig.2(a)
μ_r : reversible permeability (H/m)	Fig.2(b)
s : hysteresis parameter (Ω/m)	Fig.2(c)
A : cross-sectional area (m^2)	48.6 x 10^{-6}
C : capacitance (F)	22.5 x 10^{-6}
L : flux path length (m)	78.3 x 10^{-3}
N : number of coil turns	180
R : resistance (Ω)	1.0
E : limit of discrepancy	1.0×10^{-5}

listed in Table I, then the same period is recalculated with the modified time step width $\Delta t = \Delta t/2$. The iteration with this modification is carried out when the criterion is satisfied.

In the iterative calculation, the nonlinear parameters, shown in Fig. 2, μ and μ_r are the functions of flux density B; and s is a function of dB/dt [8].

III. RESULTS AND DISCUSSION

A. Ferroresonant Phenomenon

Fig. 4 shows the calculated results of (7) employing the parameters in listed Table I. The experimental verification of ferroresonant circuit computation employing the Chua-type magnetization model has been reported in [6]. The frequency of the driving voltage v in Fig. 4(a) is decreased from 70 to 33 Hz until time $t = 0.29$ s. Around this moment, the current i drastically increases, showing typical ferroresonance as in Fig. 4(c). This result well corresponds to those in [6].

B. Chaotic Behavior in Currents

Fig. 5 illustrates di/dt versus i obtained from Fig. 4(c), exhibiting chaos-like behavior not tracing the same locus while the frequency of the driving voltage v is fixed from $t = 0.29$ s. This is caused by hysteretic property of magnetic material.

C. Modal Analysis

In order to consider the condition of ferroresonant system, we carry out a modal analysis to the state variable equation (7) in each calculation period of the Euler method.

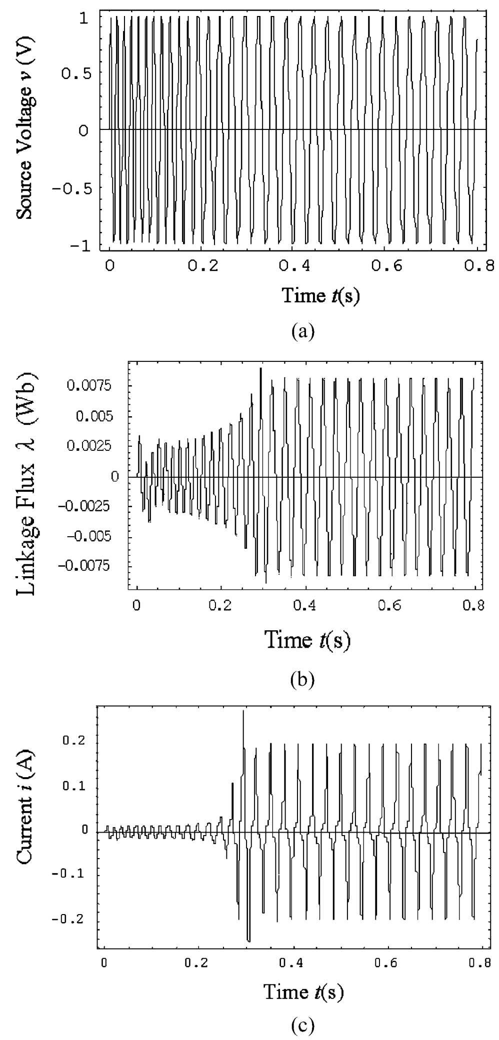


Fig. 4. Transient analysis of the ferroresonant circuit. (a) Driving voltage v. Frequency is decreased from 70 to 33 Hz until $t = 0.29$ s., then fixed. (b) Calculated linkage flux λ versus time t. (c) Calculated current i versus time t.

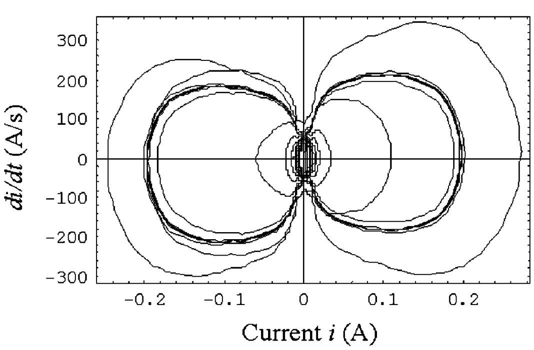


Fig. 5. Locus of di/dt versus i obtained from Fig. 4(c).

Let M be a modal matrix consisting of the characteristic vectors of matrix a as its columns, then the diagonalized state variable equation is obtained as

$$\frac{d}{dt}\hat{\mathbf{x}} = [M^{-1}aM]\hat{\mathbf{x}} + \hat{\mathbf{b}}$$
$$\mathbf{x} = M\hat{\mathbf{x}}, \quad \mathbf{b} = M\hat{\mathbf{b}} \tag{8}$$

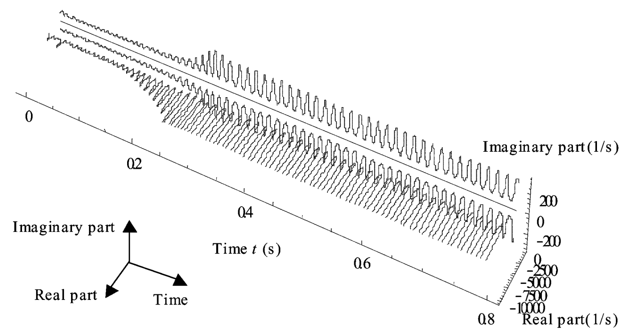


Fig. 6. Characteristic values changing of the state transition matrix a.

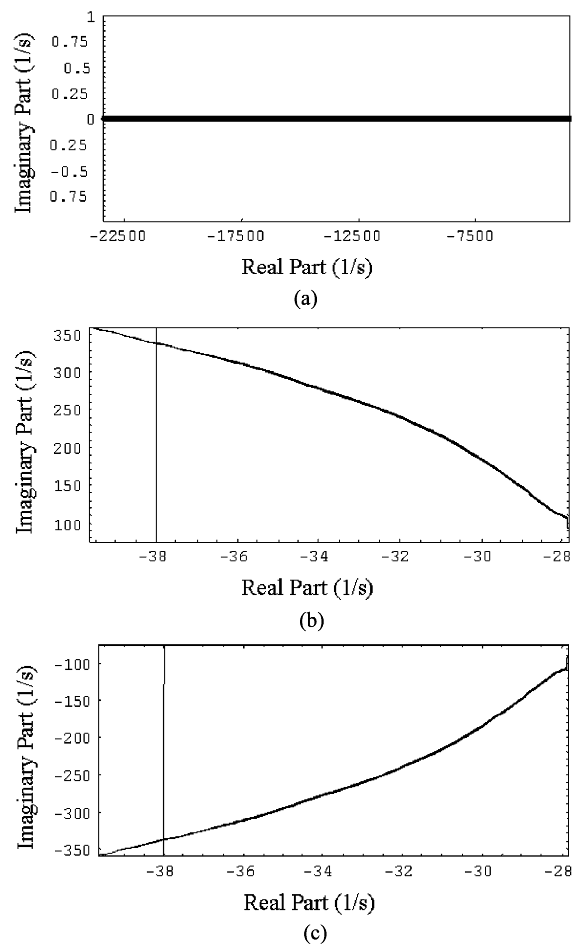
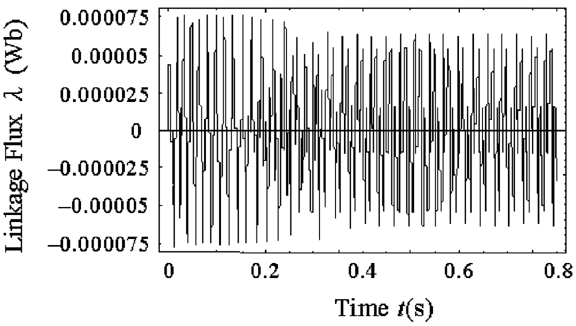


Fig. 7. Trajectories of the characteristic values derived from matrix a in (7).

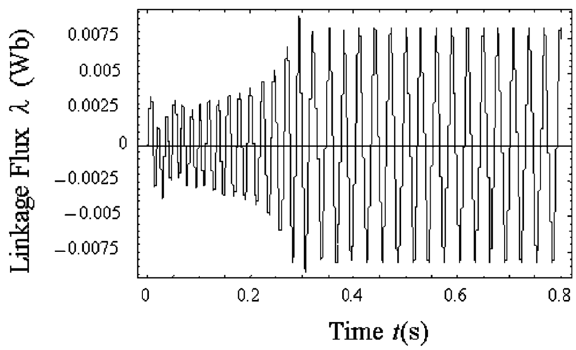
where $M^{-1}aM$ is a diagonal matrix with characteristic values. The modal analysis could be applied to the linear system so that we assume this nonlinear system to be a piecewise linear system employing fine time step widths for solving (7).

D. Trajectories of Characteristic Values

Figs. 6 and 7 show the loci of characteristic values derived by means of (8). Since a in (7) is 3×3 square matrix, then it has three characteristic values. Even though the chaotic phenomenon is exhibiting, all of the characteristic values have a regular trajectory on the left half plane, meaning that the system keeps



(a)



(b)

Fig. 8. Classified modes of the linkage flux. (a) Pure damping mode. (b) Damped oscillation mode.

in stable state. Moreover, the ferroresonant system has no hysteretic properties. Therefore, the chaotic mode is determined by the vector $\mathbf{b}$ in (7).

In the vector $\mathbf{b}$, its coefficient concerned with the right-hand side of (4) has the uncontrollable terms due to the nonlinear coefficient μ_r/s. As in (1), the coefficient μ_r/s is a significant parameter to determine the coercive force [3], [5]. Thus, it is clarified that the change of coercive force yields the ferroresonant phenomena accompanying with chaotic flicker [9], [10].

E. Mode Classification

Fig. 8 shows each mode of the linkage fluxes derived from decomposition by means of (8). Fig. 8(a) corresponds to the mode of pure real number poles shown in Fig. 7(a). Fig. 8(b) is reproduced by the mode of complex number poles of Figs. 7(b) and (c). It is obvious that Fig. 8(b) is dominated in amplitude, however, Fig. 8(a) is antiphase to Fig. 8(b). It suggests that the mode concerned with pure real number poles affects the ferroresonant mode to keep itself.

As shown in Fig. 9, the time constant varies depending on the condition of system, i.e., the matrix a depends on B and dB/dt. Observing the time constant and frequency makes it possible to predict the ferroresonance rousing.

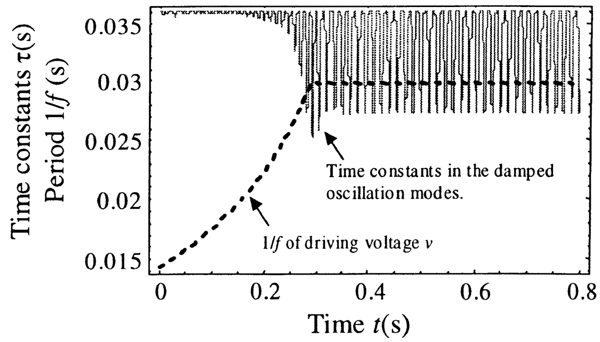


Fig. 9. Time constant changing in the damped oscillation mode.

IV. CONCLUSION

In this paper, we have studied the dynamics of series ferroresonant circuit exhibiting chaotic phenomenon. We have derived the circuit equations from the Chua-type magnetization model, and then the transient analysis has been carried out. The modal analysis of the circuit equations has also been carried out in every calculation period of the Euler method, clarifying that the ferroresonant system works in stable state although the chaotic flicking is accompanying.

As shown above, the Chua-type magnetization model enables us to classify the nonlinear system into two typical modes of dynamics. This is because the parameters of the Chua-type magnetization model are not affected by the past magnetization histories like Preisach models, etc.

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