A Weighted Inverse Matrix Approach to Searching for the Electric Field Sources

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***Abstract*—In this paper, we propose an approach to solving for an inverse problem. We try to evaluate a unique solution of an inverse problem by means of the weighted inverse matrix. We have applied our weighted inverse matrix method to searching for the electric field source and Demonstrating the three dimensional electric field distributions. As a result, we have succeeded in evaluating the unique electric field source and in visualizing the three dimensional electric vector field distributions from the locally measured electric fields.**

***Index Terms*—Fourier analysis, inverse problems, visualization, weighted inverse matrix method.**

I. Introduction

In order to avoid the mutual action and interaction among the electronic devices, electric field source searching is one of the extremely important problems in electronic device design as well as electromagnetic compatibility (EMC) problems. Searching for the electric field source from the locally measured electric fields is essentially reduced into solving for the inverse problems. In the other words, searching for the electric field source confronts to solving for an ill posed linear system [1], [2]. Various numerical method have been proposed for solving the ill posed linear systems. In biomagnetic field problems, the minimum norm methods are widely used [3]–[5]. However, the solution does not always correspond to a physically existing solution. To overcome this difficulty, we propose the new technique called weighted inverse matrix method.

In the present paper, we try to evaluate a unique solution of a electric field source searching problem by means of the weighted inverse matrix. As a result, we have succeeded in evaluating the unique electric field source distribution corresponding well to the physically existing field source.

A. System Equation

Generally, an inverse problem is reduced into solving for a following system of equations:

$$\begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ y_n \end{pmatrix} = \begin{pmatrix} G_{11} & G_{11} & \cdot & G_{1m} \\ G_{21} & G_{22} & \cdot & G_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ G_{n1} & G_{m2} & \cdot & G_{nm} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ x_m \end{pmatrix}, \quad m > n \quad \text{or}$$

$$\mathbf{Y} = C\mathbf{X} \tag{1}$$

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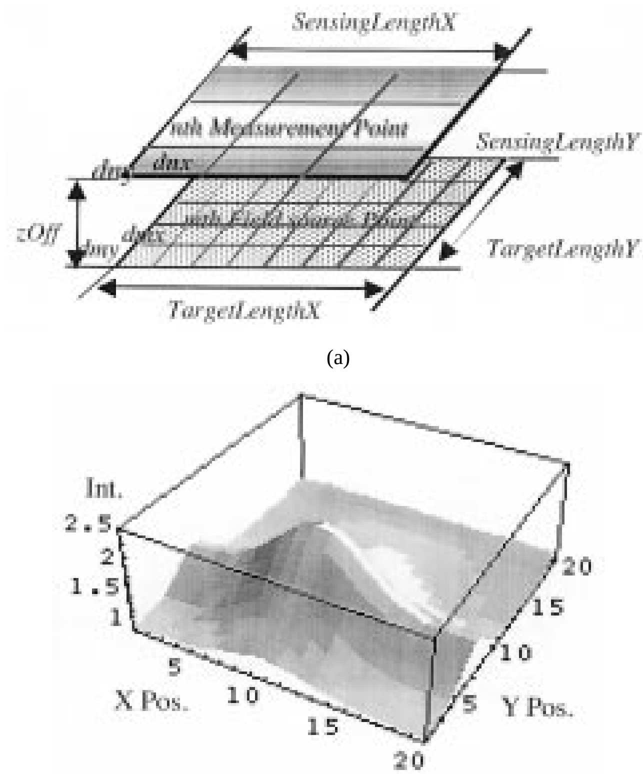


(b)

Fig. 1. Electric field source searching problem. (a) Model description; (b) model source distribution.

where $\mathbf{Y}, \mathbf{X}, C$ are the measured field vector with order n, source vector with order m, and n by m rectangular system matrix, respectively. The elements of system matrix C depend on the Green's function of physical system.

The weighted inverse matrix solution of Eq. (1) is given by

$$\mathbf{X} = W[CW]^{-1}\mathbf{Y} \tag{2}$$

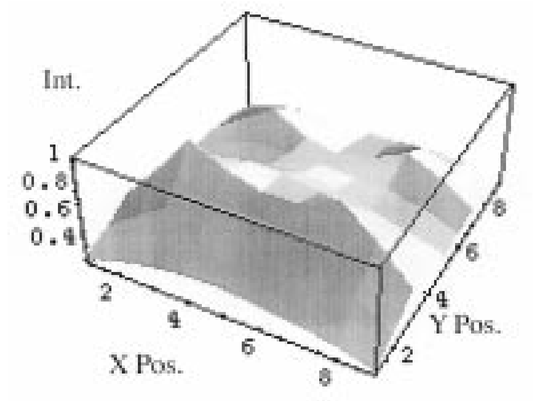
where W is a weighted matrix.

An approach when setting the condition of $W = C^T$ is the minimum norm method. Thereby, the minimum norm solution is given by Eq. (3) [6]:

$$\mathbf{X} = C^T[CC^T]^{-1}\mathbf{Y}. \tag{3}$$

B. Base Function

In the weighted inverse matrix strategy, one of the most important problems is to select the best base function. In the present paper, we employ the Fourier series as a base function. Namely, it is assumed that the solution can be represented by the Fourier series. Thus, denoting Δx and Δy as a stepwidths



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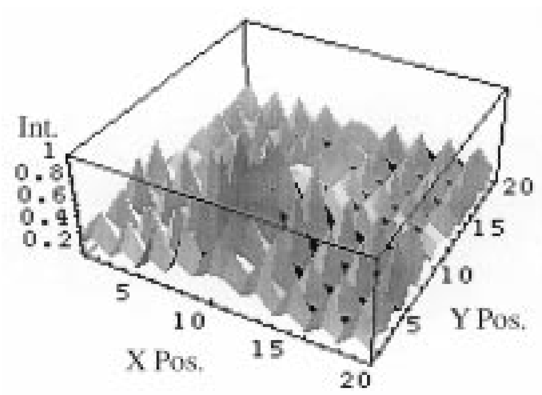


(b)

Fig. 2. Measured field intensity and minimum norm solution.

of the target space in x and y directions, the base function and weighted matrix are respectively given by (4) at the bottom of the page.

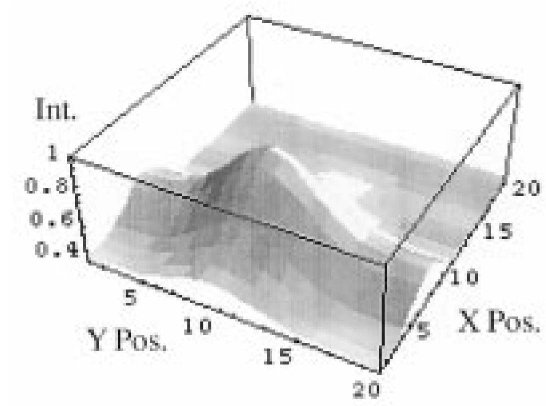
II. Electric Field Source Searching

A. Green Function

Let us consider a two dimensional inverse problem shown in Fig. 1. The elements of the system matrix C in this case are determined by

$$G = \frac{1}{r_{ij}^2}, \tag{5}$$

where r_{ij} is a distance between the measured field i $(=1, 2, \ldots, n)$ and source point j $(=1, 2, \ldots, m)$.



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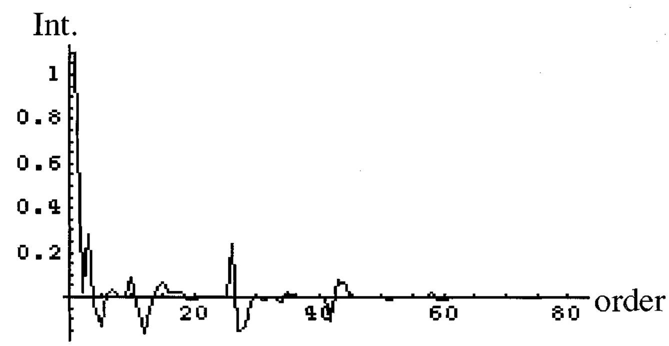

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Fig. 3. (a) Weighted inverse solution and (b) the coefficients of Fourier series.

B. Weighted Inverse Solution

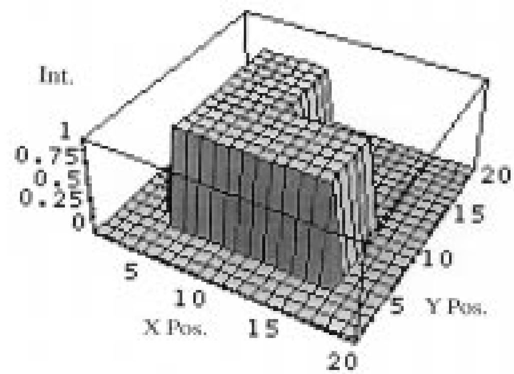
Figs. 1(a) and (b) are the problem description and an exact model of the electric field source, i.e. charge density, with order $m = 20 \times 20$, respectively.

Fig. 2(a) shows a measured electric field distribution with order $n = 9 \times 9$. Thereby, this problem is to solve a linear system having 81 equations and 400 unknowns. An application of the minimum norm method (3) to this problem yields a result shown in Fig. 2(b).

Thus, by considering the result in Fig. 2(b), conventional minimum norm solution does not always correspond to a physically existing solution. Therefore, we try to evaluate a unique solution by the weighted inverse matrix method. Fig. 3(a) shows the weighted inverse solution, also Fig. 3(b) shows the coefficients of Fourier series in this solutions. If the coefficients converge to zero, then the solution was computed successfully. As shown in

$$\begin{aligned} X(x,y) = W \cdot s = {} & s_0 + s_1 \cos x + s_2 \sin x + s_3 \cos y + s_4 \sin y + s_5 \cos x \cos y + s_6 \cos x \sin y \\ & + s_7 \sin x \cos y + s_8 \sin x \sin y + \cdots, \end{aligned}$$

$$\mathbf{X} = W \cdot \mathbf{s} = \begin{pmatrix} 1 & 1 & 0 & 1 & \cdot \\ 1 & \cos\Delta x & \sin\Delta x & \cos\Delta y & \cdot \\ 1 & \cos 2\Delta x & \sin 2\Delta x & \cos 2\Delta y & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cos(m-1)\Delta x & \sin(m-1)\Delta x & \cos(m-1)\Delta y & \cdot \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ \cdot \\ s_{n-1} \end{pmatrix}. \tag{4}$$



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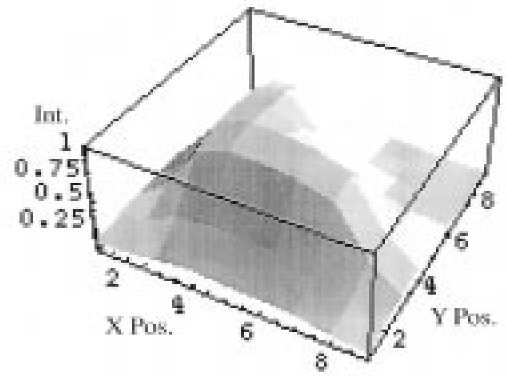

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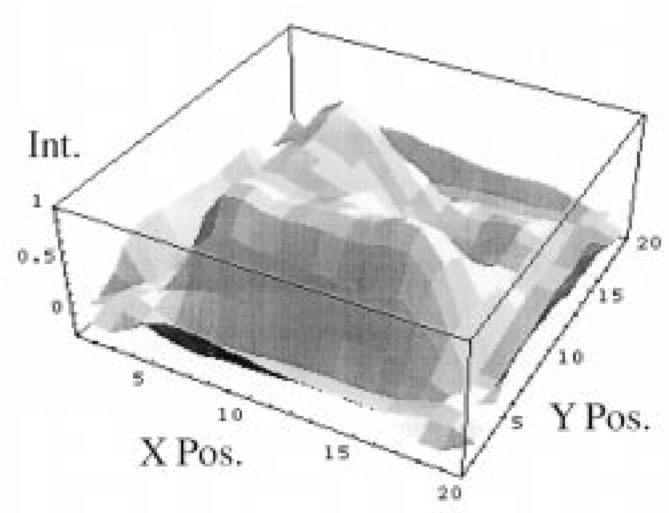

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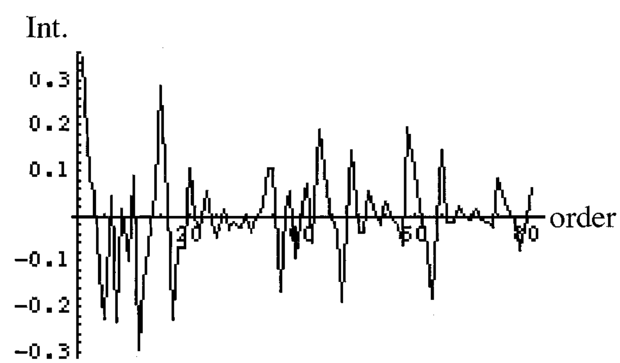
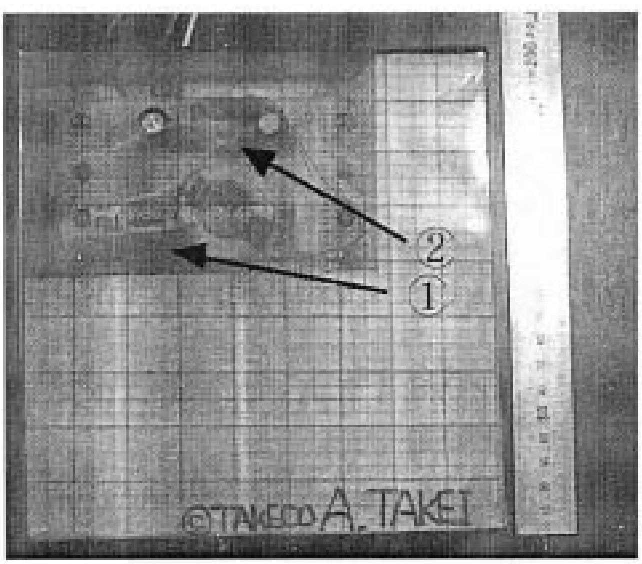


(d)

Fig. 4. (a) Model Source distribution; (b) Measured field intensity; (c) weighted inverse solution and (d) the coefficients of Fourier series.



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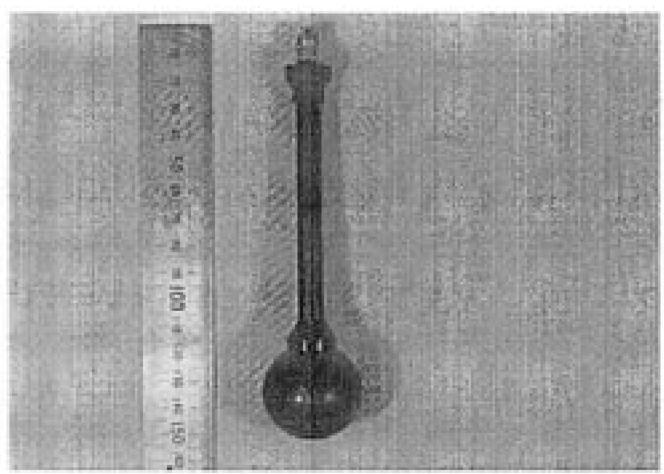
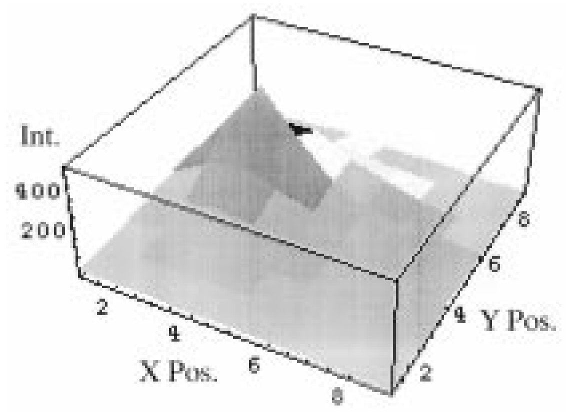


(b)

Fig. 5. (a) Experimentally used circuit board of DC/DC converter and (b) electric field probe.

Fig. 3(b), good convergence of the Fourier coefficients has been obtained [7]. As a result, a good agreement between the computed and exact solutions is obtained.

Fig. 4 shows an another example. Figs. 4(a) and (b) show an exact electric field source distribution and a measured electric field distribution, respectively. Fig. 4(c) shows a computed field



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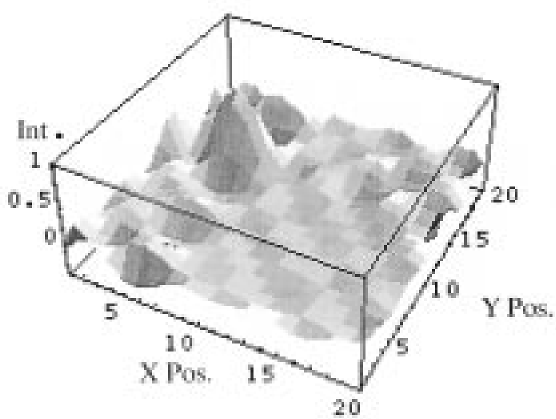

(b)

Fig. 6. (a) Measured electric field intensities and (b) weighted inverse solution.

source distribution. The electric fields were measured at equi-spaced 81 (9×9) locations on a parallel surface to target area. Also, the field source distribution was computed at equi-spaced 400 (20×20) subdivisions in the two-dimensional target area.

Even if the locally measured electric field distribution is discontinuous, the weighted inverse matrix method gives an excellent approximately solution.

Thus, we have successfully computed the field source distributions from the locally measured electric fields.

III. Leakage Electric Field Source Searching

A. Experimental

Fig. 5 shows an electronic circuit board of the target and a probe for electric field intensity sensing. The target electronic circuit consists of the MOSFET denoted by ① and diode denoted by ② that comprise a fly back type DC/DC converter.

B. Weighted Inverse Solution and Three Dimensional Electric Field

The electric fields were measured at equi-spaced 81 (9×9) locations on a parallel surface 50×50 mm^2 using an oscilloscope. The electric fields were measured on the surface of 5 mm height from the target. The operating current switched at 500 kHz is flowing through the MOSFET. Fig. 6(b) shows the electric field source distribution computed from the electric field intensities shown in Fig. 6(a). The field source in the equi-spaced 400 (20×20) subdivisions have been computed by the weighted inverse matrix method using Fourier series as a weighted base function.

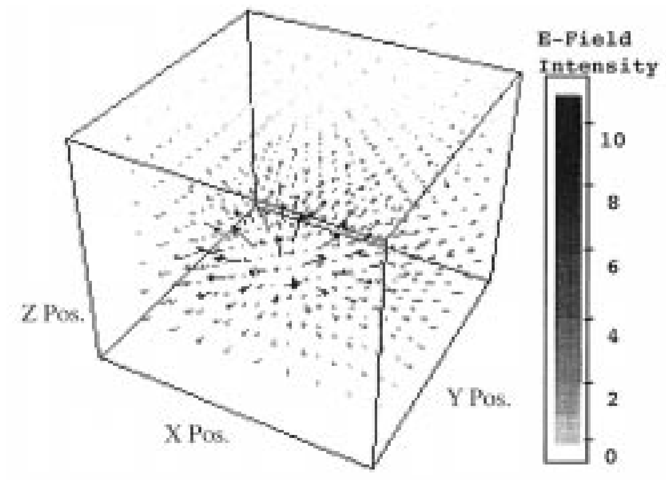

Fig. 7. Three dimensional electric field vector distributions.

The result in Fig. 6(b) has suggested that the major electric field source exists at the MOSFET.

Further, we have tried to evaluate the three dimensional electric field distributions from the weighted inverse solution. Fig. 7 shows a three dimensional electric field vector distribution.

Thus, the result in Fig. 7 has suggested that the weighted inverse matrix strategy may become one of the promizing methodologies for the EMC problems.

IV. Conclusion

In this paper, we have proposed the weighted inverse matrix method to searching for the electric field source, and tried to evaluate a unique solution. As a result, we have succeeded in evaluating the unique electric field source distribution corresponding well to the physically existing field source.

Further, we have succeeded in visualizing the three dimensional electric field vector distribution. Thus, the weighted inverse matrix method proposed in this paper is a promizing methodology for the inverse source problems.

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