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The Theory of the Harmonics of the *m,n*-Symmetrical-Machine

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Abstracts: This paper describes the new theory of the higher harmonics due to the magnetomotive force (MMF) distribution of the m-phase squirrel cage induction machine. The analytic method of this paper is the tensor analysis, using the m- and n-phase symmetrical coordinate matrices. The mathematical model of this paper is described in the following way that the stator circuits have the m-phase symmetrical winding which is connected with the star connection, and the rotor circuits have the n-phase symmetrical winding which consists of the bars. The impedance matrix of this model is transformed into the symmetrical coordinate by the m- and n-phase symmetrical coordinate matrices and its results are classified into the four cases by the relation of the number of rotor bars or phases and the pair of poles.

1. Introduction

The study about the abnormal phenomena due to the space harmonics produced by the MMF distribution and the slot permeance variation of the machine's air gap are described by several other papers. Kron pointed out the slot combinations, using the permeance wave and the electromagnetic noise due to the space harmonics. That very important problem is not yet solved perfectly in studies by Alger, Jordan and Erdelyi [1—4].

This paper is an attempt to clarify the new nature of the MMF harmonics of the m-phase squirrel cage induction machines which is the characteristic of the mutual action of the MMF space harmonics waves that plays the important role of the synchronous torque and the high frequency current of the stator circuit related with the number of the rotor bars and pair of poles.

2. Theory

2.1. Fundamental Impedance Matrix

The magnetic field due to the stepwise MMF distribution of the m-phase and $2\,p$-pole stator circuit, is written in the following eq. (1)

$$H_1 = \sum_{\nu} H_1(\nu)\,. \tag{1}$$

The magnetic field due to the saw wave MMF distribution of the n phases or bars of the rotor circuit (figure 1) is written in the following eq. (2)

$$H_2 = \sum_{\nu} H_2(\mu)\,. \tag{2}$$

The kinetic energy term of Lagrange equation is calculated from eq. (1) and eq. (2) and Rayleigh's dissipative function can be written as the electrical resistance power loss in each coordinate. Therefore, the Euler-Lagrange equation is written:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} + \frac{\partial F}{\partial \dot{x}_i} = f_i(t)\,. \tag{3}$$

The fundamental equation is derived by eq. (3) and the fundamental impedance matrix is written by the following eq. (4)

$$[\boldsymbol{Z}] = \left[\begin{array}{c|c} [\boldsymbol{Z}_{11}] & [\boldsymbol{M}_{12}] \\ \hline [\boldsymbol{M}_{21}] & [\boldsymbol{Z}_{22}] \end{array}\right]. \tag{4}$$

In eq. (4) $[\boldsymbol{Z}_{11}]$, $[\boldsymbol{Z}_{22}]$, $[\boldsymbol{M}_{12}]$, $[\boldsymbol{M}_{21}]$ are

$$[\boldsymbol{Z}_{11}] = \begin{bmatrix} R_1 + \dfrac{\mathrm{d}}{\mathrm{d}t}(\sum\limits_{\nu} L_\nu + l_1) & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\nu} L_\nu \cos\dfrac{2\,\pi}{m} & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\nu} L_\nu \cos\dfrac{m-1}{m}\,2\,\pi\,\nu \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\nu} L_\nu \cos\dfrac{2\,\pi}{m} & R_1 + \dfrac{\mathrm{d}}{\mathrm{d}t}(\sum\limits_{\nu} L_\nu + l_1) & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\nu} L_\nu \cos\dfrac{m-2}{m}\,2\,\pi\,\nu \\ \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\nu} L_\nu \cos\dfrac{m-1}{m}\,2\,\pi\,\nu & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\nu} L_\nu \cos\dfrac{m-2}{m}\,2\,\pi\,\nu & R_1 + \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\nu}(L_\nu + l_1) \end{bmatrix}$$

$$[\boldsymbol{Z}_{22}] = \begin{bmatrix} R_2 + \dfrac{\mathrm{d}}{\mathrm{d}t}(\sum\limits_{\mu} L_\mu + l_2) & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\mu} L_\mu \cos\dfrac{1}{n}\,2\,\pi\,\mu & . & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\mu} L_\mu \cos\dfrac{n-1}{n}\,2\,\pi\,\mu \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\mu} L_\mu \cos\dfrac{1}{n}\,2\,\pi\,\mu & R_2 + \dfrac{\mathrm{d}}{\mathrm{d}t}(\sum\limits_{\mu} L_\mu + l_2) & . & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\mu} L_\mu \cos\dfrac{n-2}{n}\,2\,\pi\,\mu \\ \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot & . & \cdot\;\cdot\;\cdot \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\mu} L_\mu \cos\dfrac{n-1}{n}\,2\,\pi\,\mu & \dfrac{\mathrm{d}}{\mathrm{d}t}\sum\limits_{\mu} L_\mu \cos\dfrac{n-2}{n}\,2\,\pi\,\mu & . & R_2 + \dfrac{\mathrm{d}}{\mathrm{d}t}(\sum\limits_{\mu} L_\mu + l_2) \end{bmatrix}$$

$[\boldsymbol{M}_{12}] = [\boldsymbol{M}_{21}]^T =$

$$= \frac{\mathrm{d}}{\mathrm{d}t}\sum_{\nu} M_{p\nu} \begin{bmatrix} \cos\nu\,\theta & \cos\nu\langle\theta + (1/n)\,2\,\pi\,p\rangle & . & \cos\nu\langle\theta + (n - 1/n)\,2\,\pi\,p\rangle \\ \cos\nu\langle\theta - (1/m)\,2\,\pi\rangle & \cos\nu\langle\theta - (2\pi/m) + (1/n)\,2\pi\,p\rangle & . & \cos\nu\langle\theta - (2\,\pi/m) + (n - 1/n)\,2\,\pi\,p\rangle \\ \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot & . & \cdot\;\cdot\;\cdot \\ \cos\nu\langle\theta - (m - 1/m)\,2\,\pi\rangle & \cos\nu\langle\theta - (m - 1/m)\,2\,\pi + (1/n)\,2\,\pi\,p\rangle & & \cos\nu\langle\theta - (m - 1/m)\,2\,\pi + (n - 1/n)\,2\,\pi\,p\rangle \end{bmatrix}$$

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2.2. Coordinate Transformation

The coordinate transformation is done by the m-phase and the n-phase symmetrical coordinate matrix, respectively. The stator and the rotor impedance matrix and the mutual inductance matrix are transformed by the m-phase and the n-phase symmetrical coordinate matrix.

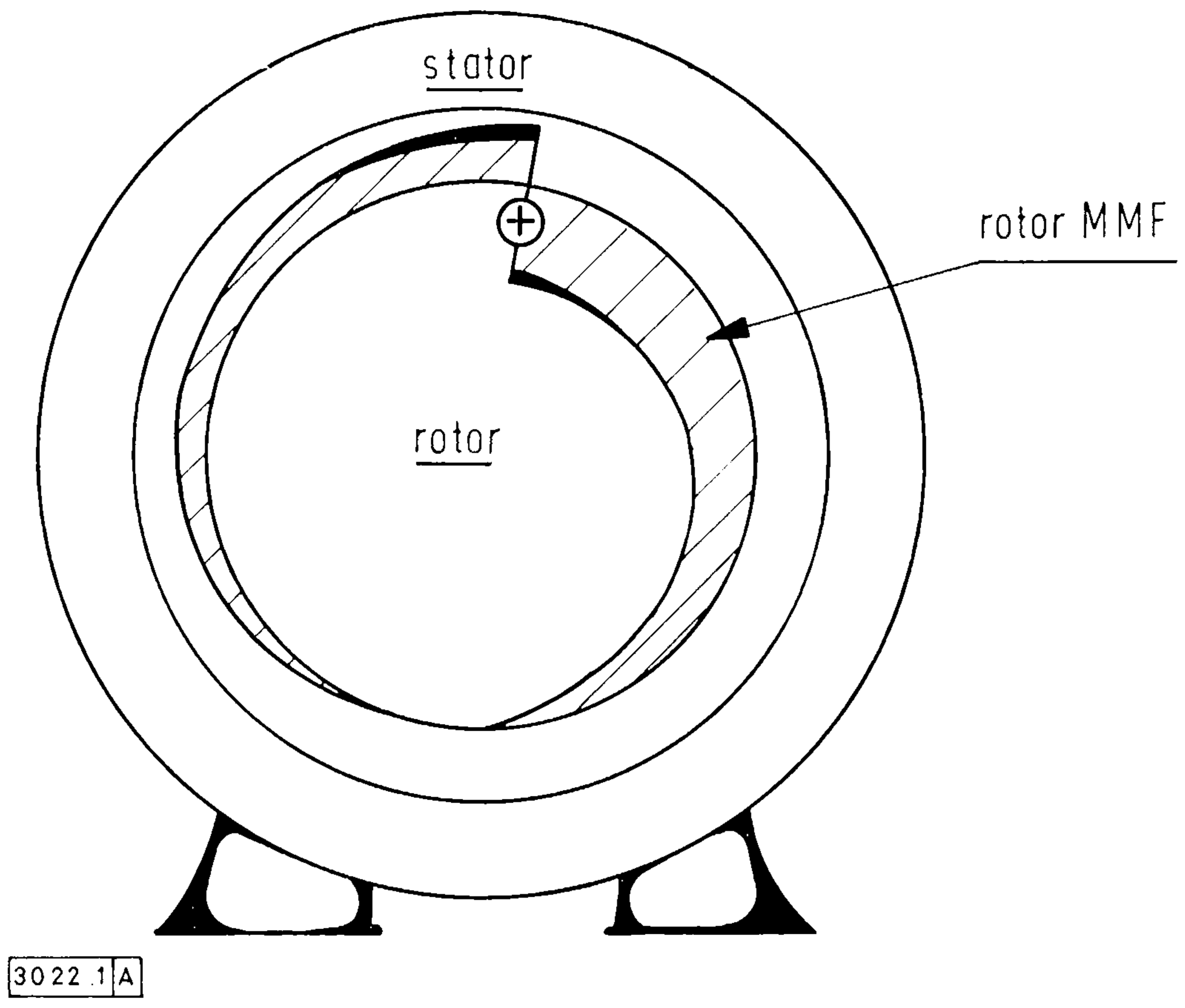

Figure 1. Rotor MMF distribution of the one bar.

The following relations between the space harmonics produced by the stator or rotor MMF distribution, and the m-phase or n-phase symmetrical coordinate matrix operator are established.

$$\alpha^{\nu} = \alpha^{2mh\pm 1} = \alpha^{\pm 1} \tag{5}$$

$$\beta^{\mu} = \beta^{gn\pm\gamma} = \beta^{\pm\gamma} \tag{6}$$

The coordinate transformation is carried out by the following equation, using eq. (5) and (6):

$$[\boldsymbol{Z}'] = \begin{bmatrix} [\boldsymbol{m}]\,[\boldsymbol{Z}_{11}]\,[\boldsymbol{m}]^{-1} & [\boldsymbol{m}]\,[\boldsymbol{M}_{12}]\,[\boldsymbol{n}]^{-1} \\ [\boldsymbol{n}]\,[\boldsymbol{M}_{21}]\,[\boldsymbol{m}]^{-1} & [\boldsymbol{n}]\,[\boldsymbol{Z}_{22}]\,[\boldsymbol{n}]^{-1} \end{bmatrix}. \tag{7}$$

The term $[\boldsymbol{Z}_{11}'] = [\boldsymbol{m}]\,[\boldsymbol{Z}_{11}]\,[\boldsymbol{m}]^{-1}$ consists of the elements (2.2) and (m, m)

$$z_{22} = z_{m,\,m} = R_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{m}{2}\sum_{\nu} L_{\nu} + l_1\right]. \tag{7a}$$

The other elements on the diagonal line of the matrix $[\boldsymbol{Z}']$ are

$$R_1 + \frac{\mathrm{d}}{\mathrm{d}t}\, l_1 \tag{7b}$$

that are zero phase impedance of the stator circuit. All the other elements of the matrix $[\boldsymbol{Z}_{11}']$ are zero.

$$[\boldsymbol{Z}'] = \begin{bmatrix} z_{22} & 0 & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{\pm\,\mathrm{j}(g\,n+\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{\pm\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) \\ 0 & z_{22} & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{\mp\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{\mp\,\mathrm{j}(g\,n+\gamma)\,(\vartheta/p) \\ \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{\mp\,\mathrm{j}(g\,n+\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{\pm\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) & z_{\gamma+1,\,\gamma+1} & 0 \\ \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{\mp\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{\pm\,\mathrm{j}(g'\,n+\gamma)\,(\vartheta/p) & 0 & z_{\gamma+1,\,\gamma+1} \end{bmatrix} \tag{8}$$

$$[\boldsymbol{Z}'] = \begin{bmatrix} z_{22} & 0 & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{+\,\mathrm{j}(g\,n+\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'\,n-\gamma}\exp\{+\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) \\ 0 & z_{22} & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{-\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{-\,\mathrm{j}(g\,n+\gamma)\,(\vartheta/p) \\ \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{-\,\mathrm{j}(g\,n+\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{+\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) & z_{\gamma+1,\gamma+1} & 0 \\ \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{-\,\mathrm{j}(1\mp g'\,n-\gamma)\,(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g} M_{gn+\gamma}\exp\{+\,\mathrm{j}(g\,n+\gamma)\,(\vartheta/p) & 0 & z_{\gamma+1,\gamma+1} \end{bmatrix} \tag{9}$$

The term $[\boldsymbol{Z}'_{22}] = [\boldsymbol{n}]\,[\boldsymbol{Z}_{22}]\,[\boldsymbol{n}]^{-1}$ consists of the elements (1.1)

$$z_{11} = R_2 + \frac{\mathrm{d}}{\mathrm{d}t}\,(\sum_g L_{gn} + l_2) \tag{7c}$$

that are zero phase impedance of the rotor circuit and the elements $(\gamma+1, \gamma+1)$ and $(n-\gamma+1, n-\gamma+1)$ of the matrix $[\boldsymbol{Z}'_{22}]$, which are

$$z_{\gamma+1,\,\gamma+1} = R_2 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{n}{2}\sum_g L_{gn+\gamma} + l_2\right). \tag{7d}$$

All the other elements of the matrix $[\boldsymbol{Z}'_{22}]$ are zero.
The term $[\boldsymbol{M}'_{12}] = [\boldsymbol{m}]\,[\boldsymbol{M}_{12}]\,[\boldsymbol{n}]^{-1}$ consists of the element $(\delta, 1)$ of the matrix $[\boldsymbol{M}'_{12}]$

$$\frac{\sqrt{m\,n}}{2}\sum_g M_{gn}\exp\left(\pm\,\mathrm{j}\,\frac{g\,n\,\theta}{p}\right) \tag{7e}$$

that are the zero phase mutual inductances of the space harmonics of the rotor circuit. The elements $(2, \gamma+1)$ of the matrix $[\boldsymbol{M}'_{12}]$ are

$$\frac{\sqrt{m\,n}}{2}\sum_g M_{gn+\gamma}\exp\left\{\pm\,\mathrm{j}\,(g\,n+\gamma)\,\frac{\theta}{p}\right\}. \tag{7f}$$

Their conjugate quantities are in the position $(m, n+1-\gamma)$ of the matrix $[\boldsymbol{M}'_{12}]$.

The term $[\boldsymbol{M}'_{21}] = [\boldsymbol{n}]\,[\boldsymbol{M}_{21}]\,[\boldsymbol{m}]^{-1}$ is derived from $[\boldsymbol{M}'_{21}] = [\boldsymbol{M}_{12}]^{*T}$.

The transformed mutual inductance matrix $[\boldsymbol{M}'_{12}]$ or $[\boldsymbol{M}'_{21}]$ is classified into the four cases by the relation of the number of rotor phases and pole pairs. Each case of the transformed impedance matrix $[\boldsymbol{Z}']$ is expressed by the following equations without zero phase components of the stator and rotor circuit.

1. If $n(1+g+g')$ is equal to $2\,p\,m(h+h')$, the mutual inductances for the harmonics of order $p(2\,m\,h \pm 1)$ and $p(2\,m\,h' \mp 1)$ are in the same column in matrix $[\boldsymbol{M}'_{12}]$. For this case the transformed impedance matrix $[\boldsymbol{Z}']$ is given in eq. (8).

2. If $n(1+g+g')$ is equal to $2\,p\langle m(h+h')+1\rangle$, the mutual inductances for the harmonics of order $p(2\,m\,h+1)$ and $p(2\,m\,h'+1)$ are in the same column in the matrix $[\boldsymbol{M}'_{12}]$. For this case the transformed impedance matrix $[\boldsymbol{Z}']$ is given in eq. (9).

3. If $n(1+g+g')$ is equal to $2\,p\langle m(h+h')-1\rangle$, the mutual inductances for the harmonics of order $p(2\,m\,h-1)$ and $p(2\,m\,h'-1)$ are in the same column in the matrix $[\boldsymbol{M}'_{12}]$. For this case the transformed impedance matrix $[\boldsymbol{Z}']$ is given in eq. (10).

4. If $n(1+g+g')$ is not equal to $2\,p\langle m(h+h') \pm (\tfrac{1}{0})\rangle$, each harmonics mutual inductance exists independently in the matrix $[\boldsymbol{M}'_{12}]$. For this case the transformed impedance matrix $[\boldsymbol{Z}']$ is given in eq. (11).

$$[\boldsymbol{Z}'] = \begin{bmatrix}
z_{22} & 0 & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{-\mathrm{j}(g\,n+\gamma)(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{-\mathrm{j}(1\mp g'\,n-\gamma)(\vartheta/p) \\
0 & z_{22} & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{+\mathrm{j}(1\mp g'\,n-\gamma)(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{+\mathrm{j}(g\,n+\gamma)(\vartheta/p) \\
\frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{+\mathrm{j}(g\,n+\gamma)(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{-\mathrm{j}(1\mp g'\,n-\gamma)(\vartheta/p) & z_{\gamma+1,\,\gamma+1} & 0 \\
\frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_{g'} M_{1\mp g'n-\gamma}\exp\{+\mathrm{j}(1\mp g'\,n-\gamma)(\vartheta/p) & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{-\mathrm{j}(g\,n+\gamma)(\vartheta/p) & 0 & z_{\gamma+1,\,\gamma+1}
\end{bmatrix} \tag{10}$$

$$[\boldsymbol{Z}'] = \begin{bmatrix}
z_{22} & 0 & \cdot & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{\mathrm{j}(g\,n+\gamma)(\vartheta/p) & 0 \\
0 & z_{22} & \cdot & 0 & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{-\mathrm{j}(g\,n+\gamma)(h/p) \\
\cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
\frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{-\mathrm{j}(g\,n+\gamma)(\vartheta/p) & 0 & \cdot & z_{\gamma+1,\gamma+1} & 0 \\
0 & \frac{\mathrm{d}}{\mathrm{d}t}(m\,n)^{1/2}/2\sum_g M_{gn+\gamma}\exp\{+\mathrm{j}(g\,n+\gamma)(\vartheta/p) & \cdot & 0 & z_{\gamma+1,\gamma+1}
\end{bmatrix} \tag{11}$$

2.3. Current and Torque

If the stator impressed m-phase voltage is perfectly balanced, the transformed voltage matrix by the m-phase symmetrical coordinate matrix consists of the second row element $U' \exp(\mathrm{j}\omega t)$ and its conjugate value of the m-th row. The currents are calculated by the iteration method and the example of the current matrix derived from the first iterative approximation of the case (2) is written by eq. (12)

The torque is calculated from eq. (13)

$$\boldsymbol{T} = \frac{p}{4}\left\{[\boldsymbol{I}'^{*}]^{T} \left\langle \frac{\partial}{\partial\theta}[\boldsymbol{Z}']\right\rangle [\boldsymbol{I}']\right\} \tag{13}$$

and the example of the torque calculation, using eq. (9) and (12), results in the following eq. (14—19):

$$\boldsymbol{T}_1 = \sum_{g} \mathrm{j}(g\,n + \gamma)\frac{\sqrt{m\,n}}{4} M_{gn+\gamma}\, i_1^{*}\, i_{gn+\gamma} \tag{14}$$

$$\boldsymbol{T}_1' = \sum_{g'} \mathrm{j}(1 \mp g'\,n - \gamma)\frac{\sqrt{m\,n}}{4} M_{1\mp g'n-\gamma}\,(i_1^{*}\, i_{1\mp g'n-\gamma}) \tag{15}$$

$$\boldsymbol{T}_2 = \sum_{g}\sum_{g'} \mathrm{j}(g\,n + \gamma)\frac{\sqrt{m\,n}}{4} M_{gn+\gamma}\, i_1'\, i^{*}_{1\mp g'n-\gamma} \tag{16}$$

$$\boldsymbol{T}_2' = \sum_{g}\sum_{g'} \mathrm{j}(1 \mp g'\,n - \gamma)\frac{\sqrt{m\,n}}{4} M_{1\mp g'n-\gamma}\, i_1'\, i^{*}_{gn+\gamma} \tag{17}$$

$$\boldsymbol{T}_3 = \sum_{g}\sum_{g'} \mathrm{j}(g\,n + \gamma)\frac{\sqrt{m\,n}}{4} M_{gn+\gamma}\,(i_1^{*}\, i^{*}_{1\mp g'n-\gamma} - i_1'^{*}\, i^{*}_{gn+\gamma})$$

$$\exp\{-\mathrm{j}\,2\,\omega\,t + \mathrm{j}(1 + g + g')\,(n\,\vartheta/p)\} \tag{18}$$

$$\boldsymbol{T}_3' = \sum_{g}\sum_{g'} \mathrm{j}(1 \mp g'\,n - \gamma)\frac{\sqrt{m\,n}}{4} M_{1\mp g'n-\gamma}\,(i_1'\, i_{1\mp g'n-\gamma} -$$

$$- i_1\, i_{gn+\gamma}) \exp\{\mathrm{j}\,2\,\omega\,t - \mathrm{j}(1 - g + g')\,(n\,\vartheta/p)\} \tag{19}$$

and the total torque is written by $\boldsymbol{T} = \boldsymbol{T}_1 + \boldsymbol{T}_1^{*} + \boldsymbol{T}_2 + \boldsymbol{T}_2^{*} + \boldsymbol{T}_3 + \boldsymbol{T}_3^{*}$.

The examination of the current matrix eq. (12) gives the following relations, that the rotor circuit currents consists of the term due to the mutual inductance for the harmonic of order $g\,n + \gamma$ and the term due to the mutual inductance for the harmonic of order $(1 + g')\,n - \gamma$. The stator circuit currents consists of the stator impressed frequeny term and the high frequency term due to the mutual action between the harmonics of order $(g\,n + \gamma)$ and $[(1 + g')\,n - \gamma]$.

The examination of the torque terms results in the following relations that the eq. (14) and (15) are the asynchronous torques produced by the stator impressed frequency current and the rotor harmonic currents; eq. (16) and (17) are the asynchronous torques produced by the stator high frequency currents and the rotor harmonic currents. Eq. (18) and (19) are the synchronous torques effected at a particular slip $s = 1 - 2\,p/n\,[1 + g + g']$ that consist of the term due to the stator impressed frequency current and the rotor harmonic currents and the term due to the stator high frequency currents and the rotor harmonic currents.

The current matrix and the torques of the other case are calculated in the same way as the previous example and its results are described as follows: the current matrix of case (1) consists of the several different stator rotor harmonic currents; in this case torques include the locking torques.

The current matrix of case (3) has the same form as in case (2), but the stator harmonic currents are of a different order from those in case (2). Therefore, there torques include the synchronous torque in the braking region.

The current matrix of the case (4) consists of the stator impressed frequency current and the rotor harmonic currents. The torques in this case consist only of asynchronous torques.

2.4. Commutation Matrix

If the harmonics' mutual inductances in the same column and same row in the transformed mutual inductance matrix excist independently, the commutation matrix can be established except the case (1), and the example of the commutation matrix for

$$[\boldsymbol{I}'] = \begin{bmatrix} i_1 \exp(\mathrm{j}\omega\,t + \sum_{g}\sum_{g'} i_1'^{*} \exp\{-\mathrm{j}\omega\,t + \mathrm{j}(1 + g + g')\,(n\,\vartheta/p)\} \\ i_1^{*} \exp(-\mathrm{j}\omega\,t) + \sum_{g}\sum_{g'} i_1' \exp\{\mathrm{j}\omega\,t - \mathrm{j}(1 + g + g')\,(n\,\vartheta/p)\} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \sum_{g} i_{gn+\gamma} \exp\{\mathrm{j}\omega\,t - \mathrm{j}(g\,n + \gamma)\,(\vartheta/p)\} + \sum_{g'} i^{*}_{1\mp g'n-\gamma} \exp\{-\mathrm{j}\omega\,t + \mathrm{j}(1 \mp g'\,n - \gamma)\,(\vartheta/p)\} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \sum_{g} i^{*}_{gn+\gamma} \exp\{-\mathrm{j}\omega\,t + \mathrm{j}(g\,n + \gamma)\,(\vartheta/p)\} + \sum_{g'} i_{1\mp g'n-\gamma} \exp\{\mathrm{j}\omega\,t - \mathrm{j}(1 \mp n\,g - \gamma)\,(\vartheta/p)\} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{bmatrix} \tag{12}$$

$$[\boldsymbol{K}] = \begin{bmatrix} 1 & & & \\ & \sum_{g}\sum_{g'} \exp\{\mathrm{j}(1 + g + g')\,(\vartheta/p)\} & & \\ & & \sum_{g} \exp\{\mathrm{j}(g\,n + \gamma)\,(\vartheta/p)\} & \\ & & & \sum_{g'} \exp\{(1 \mp g'\,n - \gamma)\,(\vartheta/p)\} \end{bmatrix} \tag{20}$$

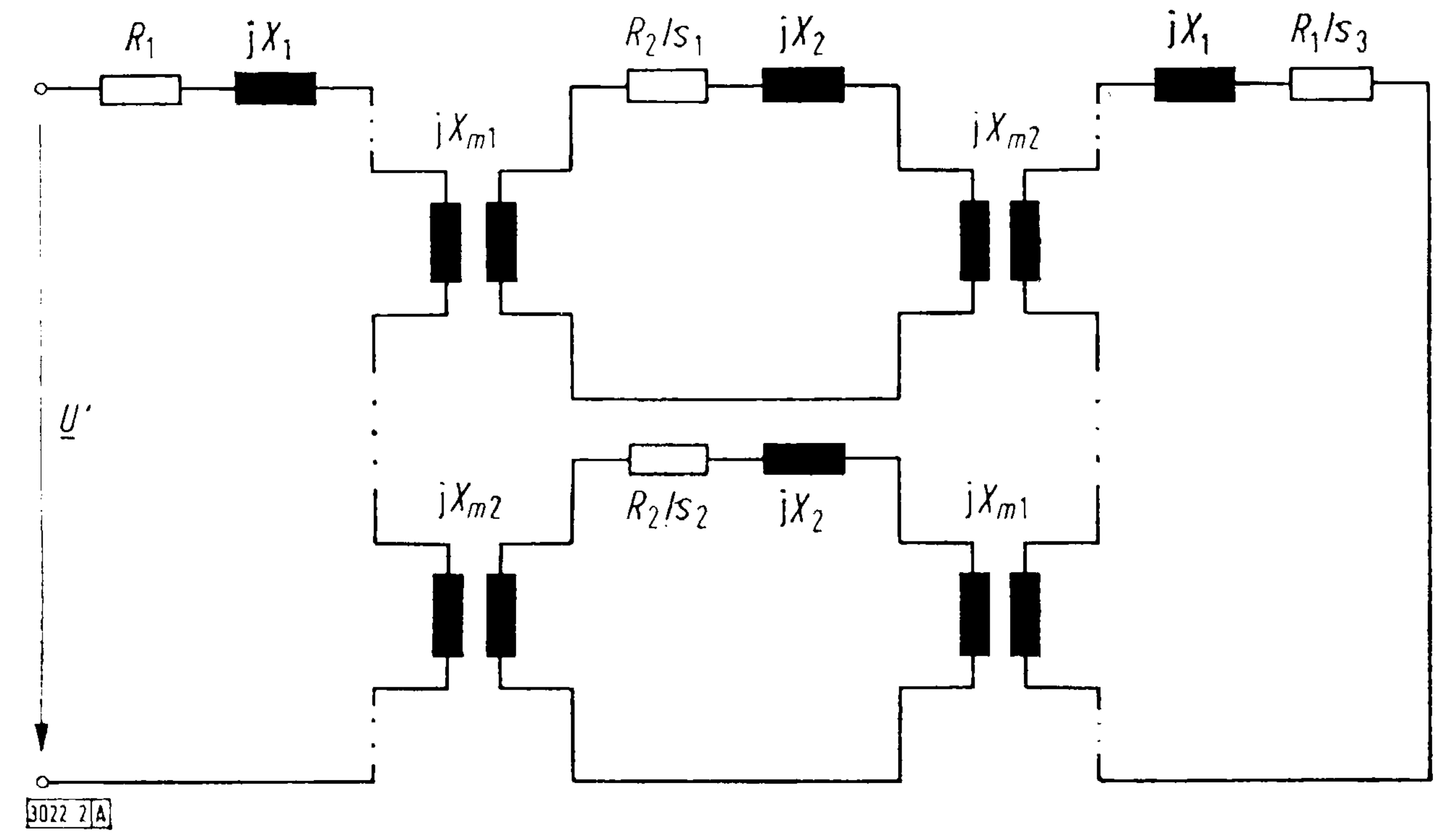

Figure 2. Steady state equivalent circuit of case (2).

$X_1 = \omega \{(m/2) \sum_{\nu} L\nu + l_1\};$ $\quad X_2 = \omega \{(n/2) \sum_{g} L_{gn+\gamma} + l_2\};$

$X_{m1} = \omega (m\, n)^{1/2}/2 \{M_{gn+\gamma}\};$ $\quad X_{m2} = \omega (m\, n)^{1/2}/2 \{M_{1 \pm g'n+\gamma}\};$

$s_1 = \langle\omega - (gn+\gamma)[(1-s)\,\omega]/p\rangle/\omega;$ $\quad s_2 = \langle\omega - (1+g'n-\gamma)(1-s)\,\omega/p\rangle/\omega$

$s_3 = \langle\omega - (1+g+g')(n/p)(1-s)\,\omega\rangle/\omega;$

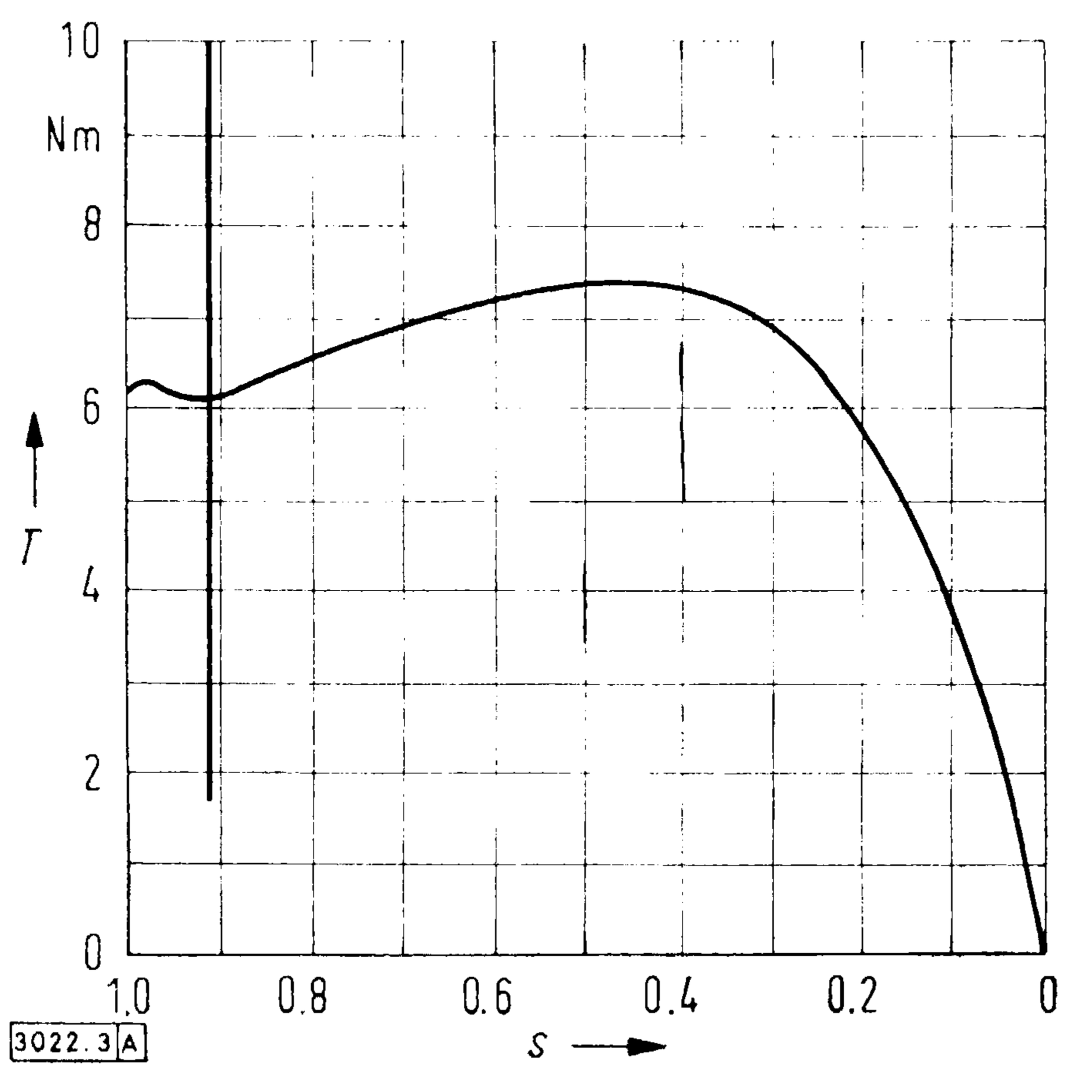

Figure 3. Torque-slip curve of case (2).

case (2) is shwon in eq. (20). The result of the commutation of the eq. (9), using eq. (20) gives the equivalent circuit for the stator transformed voltage $U' \exp(j\omega t)$, which is shown in figure 2. The numerical example of the torque-slip curve calculated by the equivalent circuit of figure 2 is shown in figure 3. Its circuit constants are given in table 2.

3. Conclusion

As shown above, I have tried to elucidate the new nature of the MMF space/harmonics of the m-phase squirrel cage induction machines modeled by the m, n-symmetrical machines.

The particular pair of the space harmonics has the mutual action in the particular sets of the number of pole pairs and the rotor phase or bars. Therefore, the particular pair of the space harmonics must be calculated simultaneously for the rigorous calculation of the phenomena due to the MMF space harmonics of the m-phase squirrel cage induction machines.

Table 1. List of Symbols and notations.

$H_1(\nu)$	magnetic field due to the stator MMF
$H_2(\mu)$	magnetic field due to the rotor MMF
L	kinetic energy term of the Lagrange function, in this case magnetic field energy
F	Rayleigh dissipative function, in this case electrical resistance power loss
$f_i(t)$	i-th coordinate external forced function, in this case stator impressed voltage
x_i	variable of the i-th coordinate, in this case electrical charge
$[\boldsymbol{Z}]$	total impedance matrix
$[\boldsymbol{Z}_{11}]$	stator impedance matrix
$[\boldsymbol{Z}_{22}]$	rotor impedance matrix
$[\boldsymbol{M}_{12}]$, $[\boldsymbol{M}_{21}]$	mutual inductance matrix
R_1, R_2	stator and rotor circuit resistance respectively
$L\nu$ $L\mu$	self-inductance of the ν-th and μ-th harmonic respectively
l_1, l_2	leakage inductance of the stator and rotor circuit respectively
$M_{p\nu}$	mutual inductance of the ν-th harmonic between the stator and rotor circuit
$[A^*]$	conjugate of matrix $[A]$
$[A']$	transformed of matrix $[A]$ in the symmetrical coordinate
$\dot{x}$	denotes the differential of x respective to time
m	number of the stator phase
n	number of the rotor phase
p	number of the pair of poles
g, g', h, h'	positive integer including zero
$\nu = 2\, m\, h \pm 1$	order of the stator MMF harmonics
μ	order of the rotor MMF harmonics, in this case positive integer
γ	order of the MMF harmonics smaller than "n"
s	slip
ω	stator impressed source frequency
θ	$(1 - s)\, \omega\, t$, angle between the stator and rotor circuit transformed into the electrical angle
α	$j\, 2\pi/m$, element of the m-phase symmetrical coordinate matrix
β	$j\, 2\pi/n$, element of the n-phase symmetrical coordinate matrix
$[\boldsymbol{m}]$, $[\boldsymbol{n}]$	m- and n-phase symmetrical coordinate matrix respectively
$[A]^T$	transpose of matrix $[A]$
$[A]^{-1}$	inverse of matrix $[A]$
δ	2 or m

Table 2. Circuit constants of calculated motor.

Number of phases	stator	$m = 3$
	rotor	$n = 40$
Number of poles		$p = 4$
Voltage (transformed value)		$U' = 200$ V
Harmonics' order ($g = g' = 0$)		$g\, n + \gamma = p$
		$(1 + g')\, n - \gamma = 19\, p$
Resistance (constant value assumed)		$R_1 = R_2 = 5\ \Omega$
Mutual reactance	p-th harmonic	$X_{m1} = 95\ \Omega$
	19 p-th harmonic	$X_{m2} = 95/(19 \times 19)\ \Omega$
Total reactance (leakage reactance included)		$X_1 = X_2 = 100\ \Omega$

4. References

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