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METHOD OF MAGNETIC CIRCUITS FOR NONLINEAR MAGNETOSTATIC FIELDS IN POLYPHASE INDUCTION MOTORS AT NO-LOAD

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The nonlinear magnetostatic field in polyphase induction motors at no-load is solved by the finite difference method based on the method of magnetic circuits.

Notation

A	amplitude of magnetomotive force (MMF)distribution [AT/m]
a	$= f(\mu)$, coefficient of finite difference equation as function of permeability
$\boldsymbol{B}$	$= \mu\boldsymbol{H}$, magnetic flux density [Wb/m^2]
b	constant determined by boundary conditions
g	mesh spacing in direction of y axis
$\boldsymbol{H}$	$= -\text{grad}\ U$, magnetic field intensity [AT/m]
h	mesh spacing in direction of x axis
M, N	number of mesh points in x and y axes, respectively
p	mesh spacing in direction of z axis
$R_i, R_{i\pm 1}$	magnetic resistances in direction of x axis at mesh points (i, j, k) and $(i \pm 1, j, k)$, respectively
$R_j, R_{j\pm 1}$	magnetic resistances in direction of y axis at mesh points (i, j, k) and $(i, j \pm 1, k)$, respectively
$R_k, R_{k\pm 1}$	magnetic resistances in direction of z axis at mesh points (i, j, k) and $(i, j, k \pm 1)$, respectively
U	scaler potential [AT]
x, y, z	rectangular coordinates
μ	$= F(\text{grad}\ U)$, permeability as function of rate of change of potential U
τ	pole pitch
ω	relaxation parameter

Superscripts K and * denote the Kth complete iterations and the first approximate value, respectively.

Subscripts $i, i \pm 1, i \pm 1/2, j, j \pm 1, j \pm 1/2, k, k \pm 1, k \pm 1/2$ refer to the positions $x_i, x_i \pm h, x_i \pm h/2, y_j, y_j \pm g, y_j \pm g/2, z_k, z_k \pm p, z_k \pm p/2$, respectively. Moreover, $m = MN$ (in two dimensions) denotes the total number of mesh points.

1. Introduction

Of all types of electric motors the polyphase induction motor is by far the most popular and the most widely used in machines. However, its space harmonic waves due to the magnetomotive force (MMF) distribution and the magnetic saturation in iron produce abnormal torques and magnetic noise.

The accurate design of polyphase induction motors is required to reduce these injurious features (e.g. [1]–[6]).

The calculation of magnetic fields is the framework of the design of all electromagnetic devices (e.g. transformers and rotating electrical machines). Because of the saturation of iron parts in electromagnetic devices it is difficult to calculate the rigorous magnetic fields analytically. To evaluate the magnetic fields in electromagnetic devices taking into account the nonlinear characteristic of iron, numerical methods are most effective. They are fundamentally divided into two classes: One is the finite difference method which replaces partial derivatives by divided differences, and the other is the finite element method which is based on variational formulations (e.g. [7]–[11]).

When we consider only the region containing iron in a certain electromagnetic device, the magnetization constant called "permeability" varies at each position with the rate of change of potential at that point. Therefore, the permeability of the region containing iron takes different values with respect to the position. On the other hand, it can be assumed that the region containing iron consists of different magnetic resistances (whose meaning will be explained later) with respect to the position, and it is possible to suppose the various connections of magnetic resistance to compose the magnetic circuits. The "method of magnetic circuits" utilizes magnetic resistance to evaluate the magnetic fields in electromagnetic devices.

The purpose of this paper is to develop the finite difference method based on magnetic circuits as a means of solving nonlinear magnetostatic fields in polyphase induction motors at no-load.

2. Finite difference equations based on magnetic circuits

This paper treats only a certain region that contains the iron and air in polyphase induction motors. In the current-free region the fundamental equations of magnetostatic fields are

$$\operatorname{div} \boldsymbol{B} = 0, \tag{1}$$

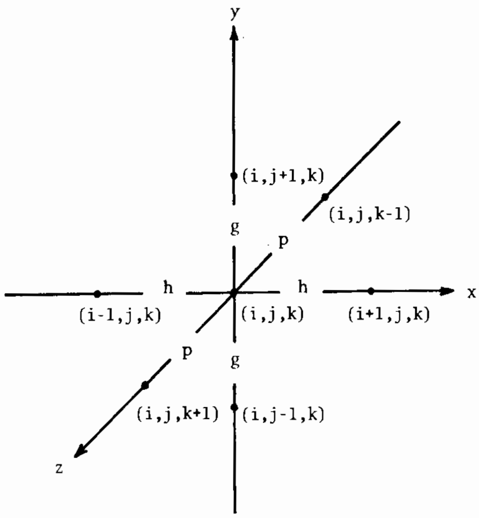
$$\boldsymbol{B} = \mu \boldsymbol{H}, \tag{2}$$

$$\boldsymbol{H} = -\operatorname{grad} U, \tag{3}$$

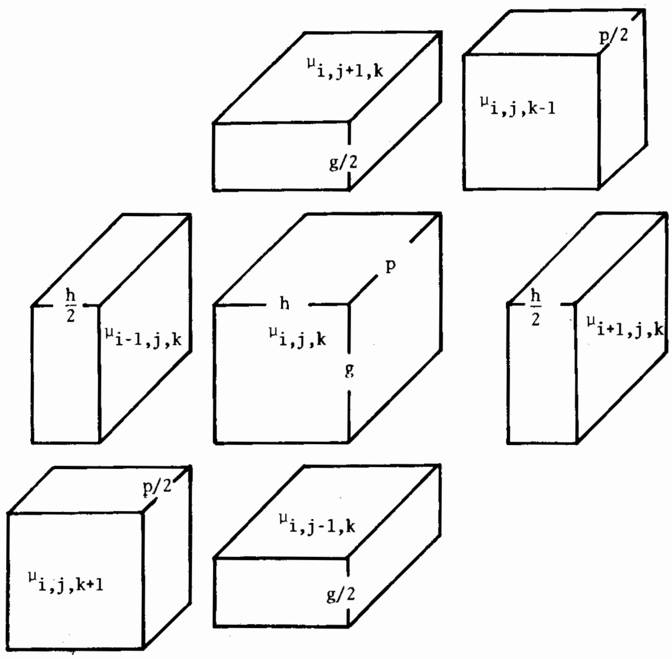
where $\boldsymbol{B}$, $\boldsymbol{H}$ and U are respectively the magnetic flux density, magnetic field intensity and scaler potential. The permeability μ is the constant value within the region containing air. However, in the region containing iron the permeability μ depends on the rate of change of the potential U at each position.

By means of eqs. (1)–(3) it is possible to write the potential equation in rectangular coordinates as

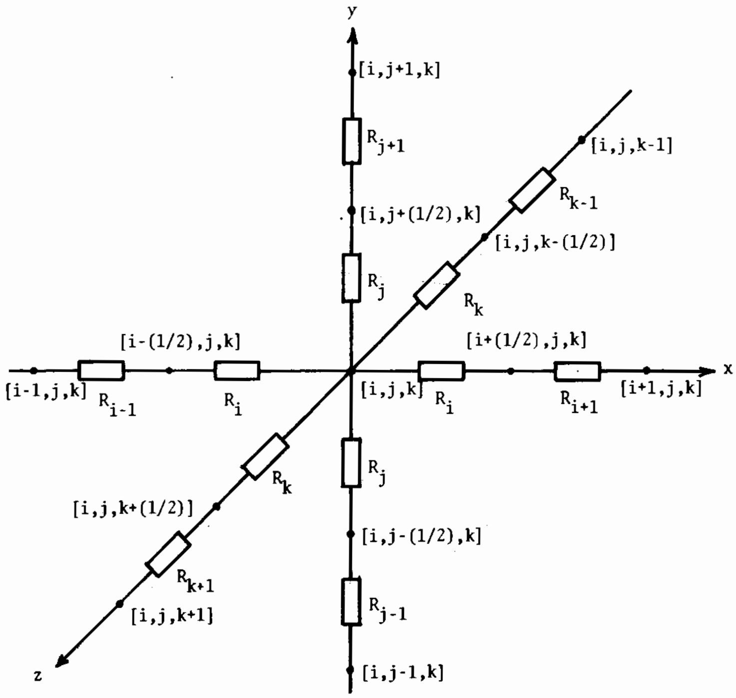
$$\frac{\partial}{\partial x}\left(\mu \frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial U}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial U}{\partial z}\right) = 0. \tag{4}$$



(a)



(b)



(c)

Fig. 1. (a) The mesh system in three-dimensional rectangular coordinates. (b) Typical subdivisions. (c) Lumped parameter representation.

In the region containing iron the permeability is represented by

$$\mu = F(\text{grad } U) = F\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right). \tag{5}$$

To deduce the finite difference representation of eq. (4), we consider the mesh system as shown in fig. 1a, where the mesh spacings are $h = x_{i+1} - x_i = x_i - x_{i-1}$, $g = y_{j+1} - y_j = y_j - y_{j-1}$, $p = z_{k+1} - z_k = z_k - z_{k-1}$.

Due to the nonlinear characteristic of iron the permeability at each mesh point takes different values with respect to the mesh points. Then as shown in fig. 1b, it is possible to consider that the region which encloses these mesh points in fig. 1a is divided into seven subdivisions in each of which the permeability may have a distinct value.

The permeabilities $\mu_{i,j,k}$, $\mu_{i+1,j,k}$, $\mu_{i-1,j,k}$, $\mu_{i,j+1,k}$, $\mu_{i,j-1,k}$, $\mu_{i,j,k+1}$ and $\mu_{i,j,k-1}$ in fig. 1b are respectively assumed to the constant values within these subdivisions. By considering the subdivisions in fig. 1b, the lumped parameter representation shown in fig. 1c is possible.

The lumped parameters in fig. 1c are the magnetic resistances, and their general definition is

$$\textit{Magnetic resistance} = \frac{\textit{Length of the magnetic flux path}}{\begin{pmatrix}\textit{Permeability of}\\ \textit{the material}\end{pmatrix} \times \begin{pmatrix}\textit{Cross-sectional area}\\ \textit{normal to the flux path}\end{pmatrix}}, \tag{6}$$

where the relations of the potential, magnetic resitance, magnetic flux, magnetic flux density and magnetic field intensity are

$$\textit{Magnetic flux} = -\frac{\textit{Potential difference}}{\textit{Magnetic resistance}}, \tag{7}$$

$$\textit{Magnetic flux density} = \frac{\textit{Magnetic flux}}{\begin{pmatrix}\textit{Cross-sectional area}\\ \textit{normal to the flux path}\end{pmatrix}}, \tag{8}$$

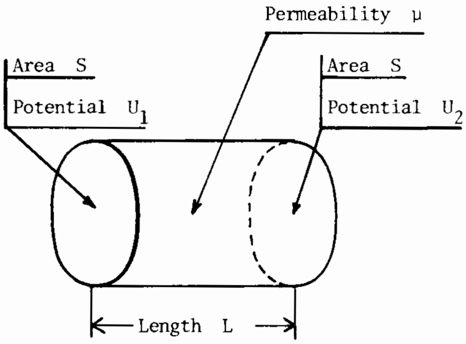
$$\textit{Magnetic field intensity} = \frac{\textit{Magnetic flux density}}{\begin{pmatrix}\textit{Permeability of}\\ \textit{the material}\end{pmatrix}}. \tag{9}$$

One of the examples of eqs. (6)–(9) is shown in fig. 2. By means of eq. (6) the magnetic resistances in fig. 1c are easily calculated as shown in table 1.

By considering eq. (2) at the mesh point (i,j,k), the first term in eq. (4) is rewritten as

$$\frac{\partial}{\partial x}\left(\mu_{i,j,k} \frac{\partial U}{\partial x} i,j,k\right) = -\frac{\partial \boldsymbol{B}}{\partial x} i,j,k. \tag{10}$$

To discretize the right-hand term of eq. (10), the partial derivative in eq. (10) is approximately replaced by the central difference, namely



Magnetic resistance $R = \frac{L}{\mu S}$

Magnetic flux $\phi = -\frac{U_1 - U_2}{R}$

Magnetic flux density $B = \frac{\phi}{S}$

Magnetic field intensity $H = \frac{B}{\mu}$

Fig. 2. The example of magnetic circuits.

$$\frac{\partial \boldsymbol{B}_{i,j,k}}{\partial x} = \frac{\boldsymbol{B}_{i+1/2,j,k} - \boldsymbol{B}_{i-1/2,j,k}}{h}, \tag{11}$$

where the subscripts $(i + 1/2, j, k)$ and $(i - 1/2, j, k)$ refer to the positions $(x_i + h/2, y_j, z_k)$ and $(x_i - h/2, y_j, z_k)$, respectively.

By means of eqs. (7), (8) the magnetic flux densities are

$$\boldsymbol{B}_{i+1/2,j,k} = -\frac{1}{gp}\left(\frac{U_{i+1,j,k} - U_{i,j,k}}{R_{i+1} + R_i}\right) = \frac{2}{h}\left(\frac{\mu_{i+1,j,k}\,\mu_{i,j,k}}{\mu_{i+1,j,k} + \mu_{i,j,k}}\right)(U_{i,j,k} - U_{i+1,j,k}) \tag{12}$$

Table 1
Magnetic resistances

x axis	$R_i = \frac{h/2}{\mu_{i,j,k}gp}$	$R_{i+1} = \frac{h/2}{\mu_{i+1,j,k}gp}$	$R_{i-1} = \frac{h/2}{\mu_{i-1,j,k}gp}$
y axis	$R_j = \frac{g/2}{\mu_{i,j,k}hp}$	$R_{j+1} = \frac{g/2}{\mu_{i,j+1,k}hp}$	$R_{j-1} = \frac{g/2}{\mu_{i,j-1,k}hp}$
z axis	$R_k = \frac{p/2}{\mu_{i,j,k}hg}$	$R_{k+1} = \frac{p/2}{\mu_{i,j,k+1}hg}$	$R_{k-1} = \frac{p/2}{\mu_{i,j,k-1}hg}$

and

$$\boldsymbol{B}_{i-1/2,j,k} = -\frac{1}{gp}\left(\frac{U_{i,j,k} - U_{i-1,j,k}}{R_{i-1} + R_i}\right) = \frac{2}{h}\left(\frac{\mu_{i-1,j,k}\mu_{i,j,k}}{\mu_{i-1,j,k} + \mu_{i,j,k}}\right)(U_{i-1,j,k} - U_{i,j,k}), \tag{13}$$

where the magnetic resistances R_{i+1}, R_i and R_{i-1} are given in table 1.

When eqs. (12), (13) are substituted into eq. (11), then eq. (10) reduces to

$$\frac{\partial}{\partial x}\left(\mu_{i,j,k}\frac{\partial U_{i,j,k}}{\partial x}\right) = \frac{2}{h^2}\mu_{i,j,k}\left[\frac{\mu_{i+1,j,k}U_{i+1,j,k}}{\mu_{i+1,j,k}+\mu_{i,j,k}} - \left(\frac{\mu_{i+1,j,k}}{\mu_{i+1,j,k}+\mu_{i,j,k}} + \frac{\mu_{i-1,j,k}}{\mu_{i-1,j,k}+\mu_{i,j,k}}\right)U_{i,j,k} + \frac{\mu_{i-1,j,k}U_{i-1,j,k}}{\mu_{i-1,j,k}+\mu_{i,j,k}}\right]. \tag{14}$$

Similarly, it can be shown that the other terms in eq. (4) are

$$\frac{\partial}{\partial y}\left(\mu_{i,j,k}\frac{\partial U_{i,j,k}}{\partial y}\right) = \frac{2}{g^2}\mu_{i,j,k}\left[\frac{\mu_{i,j+1,k}U_{i,j+1,k}}{\mu_{i,j+1,k}+\mu_{i,j,k}} - \left(\frac{\mu_{i,j+1,k}}{\mu_{i,j+1,k}+\mu_{i,j,k}} + \frac{\mu_{i,j-1,k}}{\mu_{i,j-1,k}+\mu_{i,j,k}}\right)U_{i,j,k} + \frac{\mu_{i,j-1,k}U_{i,j-1,k}}{\mu_{i,j-1,k}+\mu_{i,j,k}}\right] \tag{15}$$

and

$$\frac{\partial}{\partial z}\left(\mu_{i,j,k}\frac{\partial U_{i,j,k}}{\partial z}\right) = \frac{2}{p^2}\mu_{i,j,k}\left[\frac{\mu_{i,j,k+1}U_{i,j,k+1}}{\mu_{i,j,k+1}+\mu_{i,j,k}} - \left(\frac{\mu_{i,j,k+1}}{\mu_{i,j,k+1}+\mu_{i,j,k}} + \frac{\mu_{i,j,k-1}}{\mu_{i,j,k-1}+\mu_{i,j,k}}\right)U_{i,j,k} + \frac{\mu_{i,j,k-1}U_{i,j,k-1}}{\mu_{i,j,k-1}+\mu_{i,j,k}}\right]. \tag{16}$$

By combining eqs. (14)–(16), we can obtain the finite difference representation of eq. (4) by the method of magnetic circuits.

In the region containing air eqs. (14)–(16) reduce to conventional representations of second-order partial derivatives [14] since the permeabilities in eqs. (14)–(16) are the same values. However, in the region containing iron the permeabilities in eqs. (14)–(16) depend on the rate of change of the potential at each mesh point. For example, by means of eq. (5) the permeability at the mesh point (i, j, k) is

$$\mu_{i,j,k} = F\left(\frac{\partial U_{i,j,k}}{\partial x}, \frac{\partial U_{i,j,k}}{\partial y}, \frac{\partial U_{i,j,k}}{\partial z}\right). \tag{17}$$

The partial derivative $\partial U_{i,j,k}/\partial x$ in eq. (17) is approximately replaced by th central difference, that is

$$\frac{\partial U_{i,j,k}}{\partial x} = \frac{U_{i+1/2,j,k} - U_{i-1/2,j,k}}{h}, \tag{18}$$

where the potentials $U_{i+1/2,j,k}$ and $U_{i-1/2,j,k}$ are determined by the continuity of magnetic flux. By considering eqs. (12), (13), the magnetic flux must be continuous from mesh point $(i+1, j, k)$ to (i, j, k), and from mesh point (i, j, k) to $(i-1, j, k)$. Therefore, the following equations must be satisfied:

$$\frac{U_{i+1,j,k} - U_{i+1/2,j,k}}{R_{i+1}} = \frac{U_{i+1/2,j,k} - U_{i,j,k}}{R_i} \tag{19}$$

and

$$\frac{U_{i,j,k} - U_{i-1/2,j,k}}{R_i} = \frac{U_{i-1/2,j,k} - U_{i-1,j,k}}{R_{i-1}}. \tag{20}$$

By combining eqs. (18)–(20), we obtain

$$\frac{\partial U_{i,j,k}}{\partial x} = \frac{1}{h}\left(\frac{\mu_{i+1,j,k} U_{i+1,j,k} + \mu_{i,j,k} U_{i,j,k}}{\mu_{i+1,j,k} + \mu_{i,j,k}} - \frac{\mu_{i-1,j,k} U_{i-1,j,k} + \mu_{i,j,k} U_{i,j,k}}{\mu_{i-1,j,k} + \mu_{i,j,k}}\right). \tag{21}$$

Similarly, it is possible to show that the other terms in eq. (17) are

$$\frac{\partial U_{i,j,k}}{\partial y} = \frac{1}{g}\left(\frac{\mu_{i,j+1,k} U_{i,j+1,k} + \mu_{i,j,k} U_{i,j,k}}{\mu_{i,j+1,k} + \mu_{i,j,k}} - \frac{\mu_{i,j-1,k} U_{i,j-1,k} + \mu_{i,j,k} U_{i,j,k}}{\mu_{i,j-1,k} + \mu_{i,j,k}}\right) \tag{22}$$

and

$$\frac{\partial U_{i,j,k}}{\partial z} = \frac{1}{p}\left(\frac{\mu_{i,j,k+1} U_{i,j,k+1} + \mu_{i,j,k} U_{i,j,k}}{\mu_{i,j,k+1} + \mu_{i,j,k}} - \frac{\mu_{i,j,k-1} U_{i,j,k-1} + \mu_{i,j,k} U_{i,j,k}}{\mu_{i,j,k-1} + \mu_{i,j,k}}\right). \tag{23}$$

When eqs. (21)–(23) are substituted into eq. (17), then we can formally obtain the permeability $\mu_{i,j,k}$ as a function of the potentials and of the permeabilities.

At an air-iron boundary the tangential component of magnetic field intensity as well as the normal component of magnetic flux density must be continuous.

In the finite difference formulations the tangential component of magnetic field intensity is always continuous since an air-iron boundary is used in common. Because of the theory of magnetic circuits (e.g. eqs. (12), (13), (19) and (20)) the normal component of magnetic flux is always continuous in all regions.

The considerations for other boundary conditions are treated in much the same way as reported in [7].

3. Nonlinear magnetostatic fields in polyphase induction motors

With regard to the symmetrical mechanical structure, idealized magnetomotive force distribution and no-load condition of polyphase induction motors, the magnetic field may be solved for a modified form of the region as shown in fig. 3 (see [2] and [3]). This means that the solution can be carried out in a rectangular coordinate system without committing any appreciable error for polyphase (linear and rotating) induction motors at no-load.

Since we are dealing with the current-free region (or scaler potential problem), it is possible to divide the region (fig. 3) into two distinct regions.

One of the regions is the stator and the other is the rotor including the air gap.

Let there be M mesh points along the x axis and N mesh points along the y axis; then MN potentials must be calculated in order to numerically solve the magnetostatic field in each region. Ultimately, the calculation of MN potentials reduces to the solution of a system of MN nonlinear simultaneous algebraic equations since the finite difference equation that takes into account the nonlinear characteristic of iron is one of the nonlinear algebraic equations. This system is generally written

$$\left.\begin{array}{l} a_{1,1}U_1 + a_{1,2}U_2 + \ldots + a_{1,m}U_m = b_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m-1,1}U_1 + \ldots + a_{m-1,m-1}U_{m-1} + a_{m-1,m}U_m = b_{m-1} \\ a_{m,1}U_1 + \ldots + a_{m,m-1}U_{m-1} + a_{m,m}U_m = b_m \end{array}\right\}, \tag{24}$$

where $m = MN$; the coefficients $a_{1,1}, a_{1,2}, \ldots a_{m,m}$ depend on the permeabilities $\mu_1, \mu_2, \ldots \mu_m$; and $b_1, b_2, \ldots b_m$ are constants determined by the boundary conditions. Note that the permeabilities depend on the potentials as well as the permeabilities (cf. eqs. (17), (21), (22) and (23)). In order to calculate the potentials $U_1, U_2, \ldots U_m$, eq. (24) is solved by iteration (see [9]).

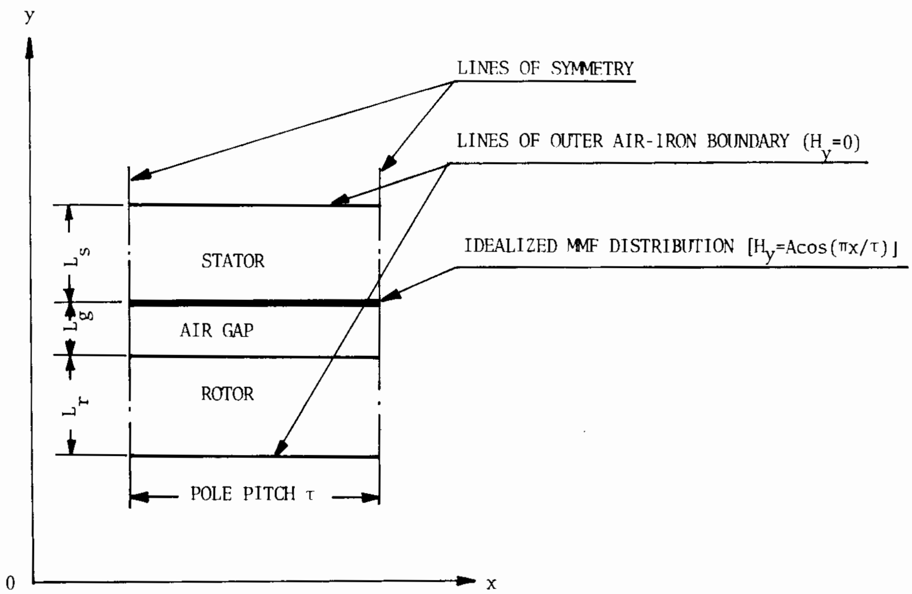


Fig. 3. Simplified cross section of polyphase induction motors at no-load.

One of the steps of this iteration method is

$$\left.\begin{aligned}
&[\mu_{m-1}^{(K)}]^* = F[U_1^{(K+1)}, \ldots U_{m-2}^{(K+1)}, U_{m-1}^{(K)}, U_m^{(K)}, \mu_1^{(K)}, \ldots \mu_{m-2}^{(K)}, \mu_{m-1}^{(K-1)}, \mu_m^{(K-1)}]\\
&\mu_{m-1}^{(K)} = \mu_{m-1}^{(K-1)} + (1/\omega)([\mu_{m-1}^{(K)}]^* - \mu_{m-1}^{(K-1)})\\
&a_{m-1,1} = f_{m-1,1}(\mu_1^{(K)}, \ldots \mu_{m-2}^{(K)}, \mu_{m-1}^{(K)}, \mu_m^{(K-1)})\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&a_{m-1,m-2} = f_{m-1,m-2}(\mu_1^{(K)}, \ldots \mu_{m-2}^{(K)}, \mu_{m-1}^{(K)}, \mu_m^{(K-1)})\\
&a_{m-1,m} = f_{m-1,m}(\mu_1^{(K)}, \ldots \mu_{m-2}^{(K)}, \mu_{m-1}^{(K)}, \mu_m^{(K-1)})\\
&[U_{m-1}^{(K+1)}]^* = -(1/a_{m-1,m-1})[a_{m-1,1}U_1^{(K+1)} + \ldots + a_{m-1,m-2}U_{m-2}^{(K+1)} + a_{m-1,m}U_m^{(K)} - b_{m-1}]\\
&U_{m-1}^{(K+1)} = U_{m-1}^{(K)} + \omega([U_{m-1}^{(K+1)}]^* - U_{m-1}^{(K)})
\end{aligned}\right\}, \qquad (25)$$

where the superscripts K and * denote respectively the Kth complete iteration and the first approximate value, and ω is the relaxation parameter.

By continuiting the iterations, it is possible to access the potentials $U_1, U_2, \ldots U_m$ which satisfy eqs. (24). Note that the potentials $U_1, U_2, \ldots U_m$ are overrelaxed, but that the permeabilities μ_1, $\mu_2, \ldots \mu_m$ are underrelaxed to suppress the variations of the coefficients $a_{1,1}, a_{1,2}, \ldots a_{m,m}$.

Various constants for the numerical example are given in table 2.

After two hundred iterations (maximum deviation is about 0.7 percent) the results of numerical solutions combined with the stator and rotor regions are shown in figs. 4 and 5.

To check the validity of these results, the potentials in fig. 4 are substituted into the finite difference equation derived by the quasilinearization method (which is described in the appendix). As a result it can be shown that the potentials calculated by the method of magnetic circuits completely satisfy the finite difference equations derived by the quasilinearization method.

Table 2
Various constants of the example (assumed)

Number of mesh points	$M = 10, N = 25$	Station region
	$M = 10, N = 26$	Rotor region
Length	$L_s = 0.025(m)$, $L_g = 0.001(m)$, $L_r = 0.025(m)$	
Pole pitch	$\tau = 0.15(m)$	
Relaxation parameter	$\omega = 1.8$	
Permeabilities	$\mu_{air} = 1.0\ (H/m)$	
	$\mu_{iron} = \dfrac{200}{1.0 + 142\sqrt{\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}}\ (H/m)$	

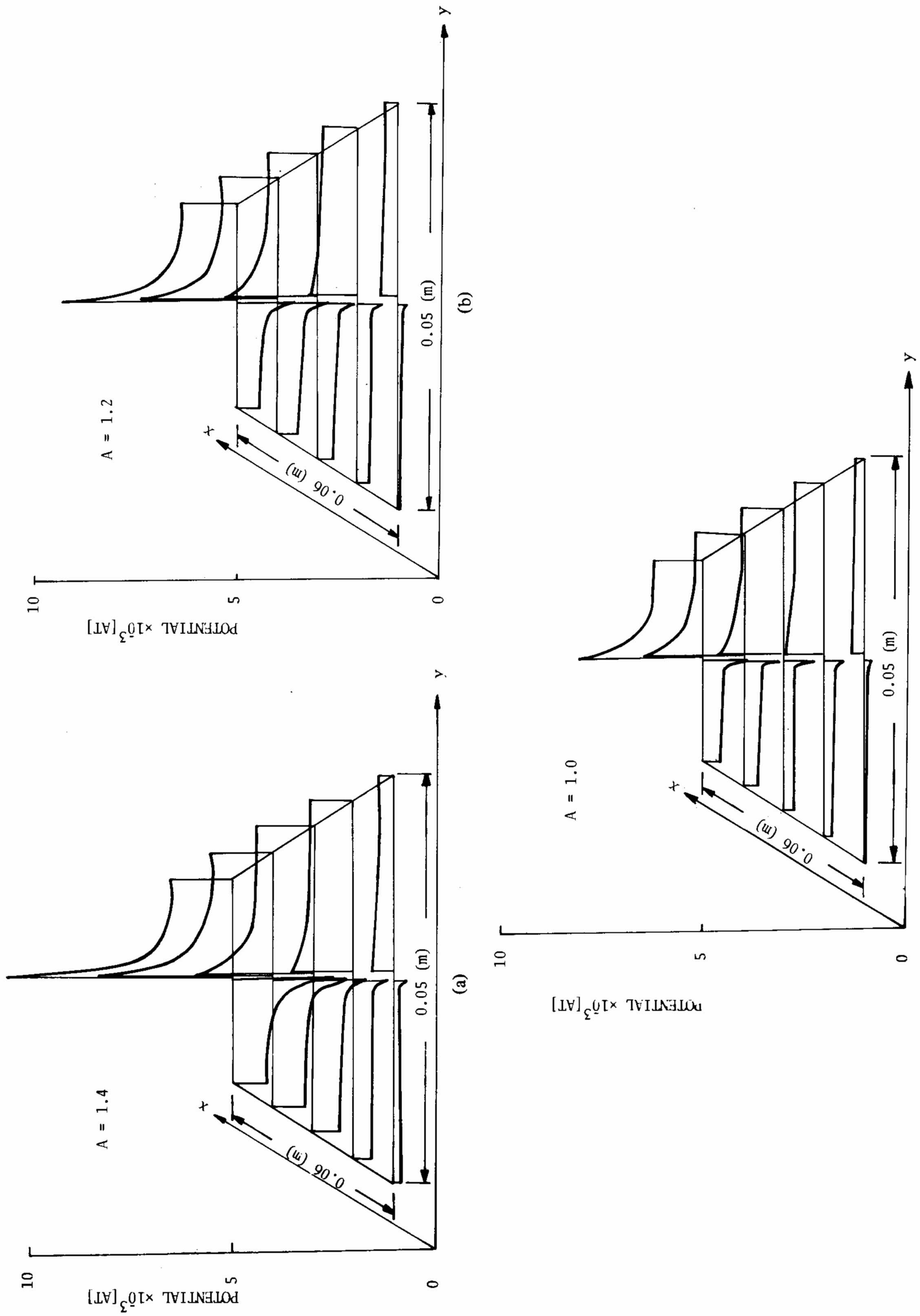
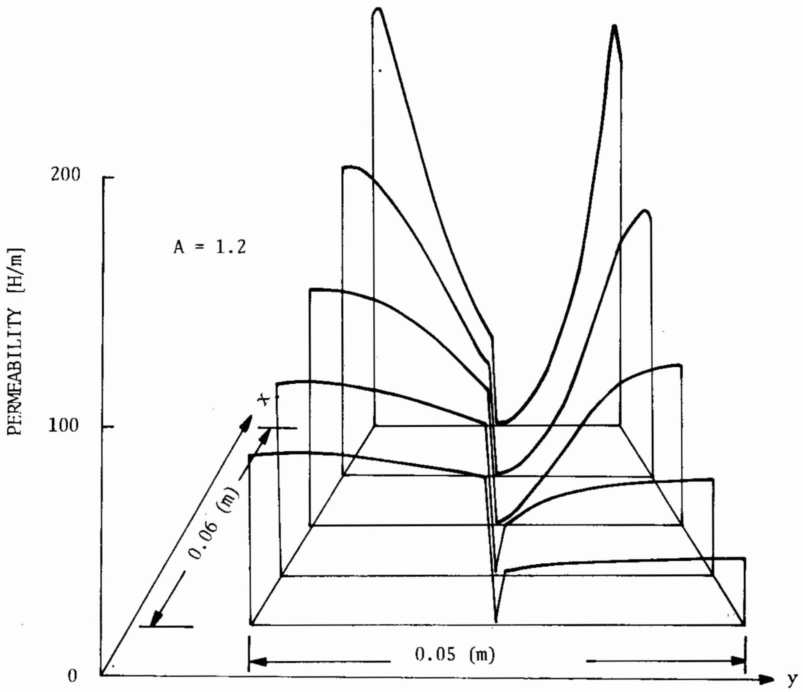
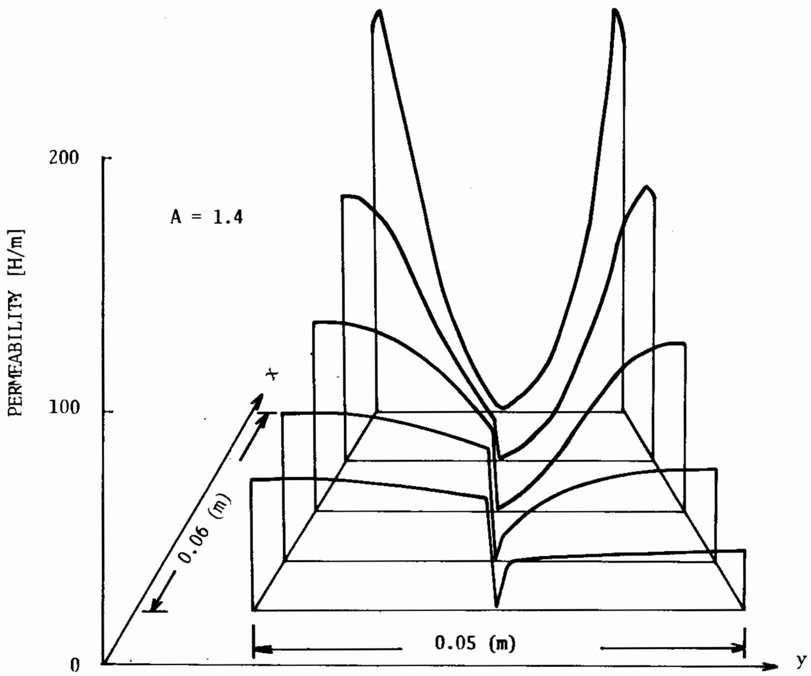


Fig. 4. Numerical solution of the potentials (the right-hand and left-hand regions are respectively the stator and the rotor regions).


200
A = 1.2
PERMEABILITY [H/m]
100
x
0.06 (m)
0
0.05 (m)
y

(5a).


200
A = 1.4
PERMEABILITY [H/m]
100
x
0.06 (m)
0
0.05 (m)
y

(5b)

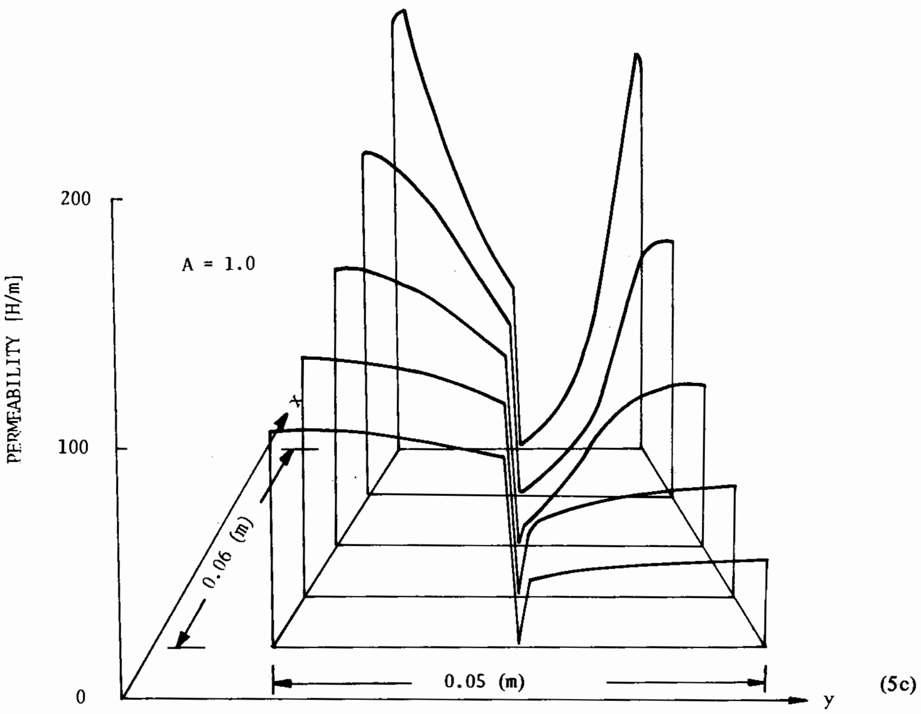


Fig. 5. Numerical solution of the permeabilities (the right-hand and left-hand regions are respectively the stator and rotor regions).

4. Conclusion

In this paper a new finite difference equation based on the method of magnetic circuits has been derived for numerically solving the nonlinear magnetostatic fields in polyphase induction motors at no-load. Consequently, it has been shown that magnetic saturation has occurred in the stator magnetic frame in polyphase induction motors at no-load.

Especially, the finite difference method based on magnetic circuits is quite a useful method for the computations of nonlinear magnetostatic fields in all the electromagnetic devices since almost all boundary conditions that arise in magnetostatic field problem are either automatically satisfied or easily handled by considering the method of subdivision.

In order to calculate the 250 potentials, it required only a few minutes for two hundred iterations on the computer FACOM 230-45S.

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The practical computations given in this paper were carried out by using the computers FACOM 230-45S of Hosei University and HITAC 8800/8700 of Tokyo University.

Appendix. Quasilinearization method

In the two-dimensional $x-y$ plane each term of eq. (4) in section 2 is approximately replaced by the following divided differences [7], [15]:

$$\frac{\partial}{\partial x}\left(\mu_{i,j}\frac{\partial U_{i,j}}{\partial x}\right) = (1/h)\left[\mu_{i+1/2,j}\frac{U_{i+1,j}-U_{i,j}}{h} - \mu_{i-1/2,j}\frac{U_{i,j}-U_{i-1,j}}{h}\right], \tag{A-1}$$

$$\frac{\partial}{\partial y}\left(\mu_{i,j}\frac{\partial U_{i,j}}{\partial y}\right) = (1/g)\left[\mu_{i,j+1/2}\frac{U_{i,j+1}-U_{i,j}}{g} - \mu_{i,j-1/2}\frac{U_{i,j}-U_{i,j-1}}{g}\right], \tag{A-2}$$

where the permeabilities are

$$\mu_{i+1/2,j} = F\left(\frac{U_{i+1,j}-U_{i,j}}{h}, \frac{U_{i+1,j+1}-U_{i+1,j-1}+U_{i,j+1}-U_{i,j-1}}{4g}\right), \tag{A-3}$$

$$\mu_{i-1/2,j} = F\left(\frac{U_{i,j}-U_{i-1,j}}{h}, \frac{U_{i,j+1}-U_{i,j-1}+U_{i-1,j+1}-U_{i-1,j-1}}{4g}\right), \tag{A-4}$$

$$\mu_{i,j+1/2} = F\left(\frac{U_{i+1,j+1}-U_{i-1,j+1}+U_{i+1,j}-U_{i-1,j}}{4h}, \frac{U_{i,j+1}-U_{i,j}}{g}\right), \tag{A-5}$$

$$\mu_{i,j-1/2} = F\left(\frac{U_{i+1,j}-U_{i-1,j}+U_{i+1,j-1}-U_{i-1,j-1}}{4h}, \frac{U_{i,j}-U_{i,j-1}}{g}\right). \tag{A-6}$$

By combining eqs. (A-1)–(A-6), it is possible to derive the finite difference equations of eq. (4) in the two-dimensional x-y plane.

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