A formulation of the inverse problems in magnetostatic fields and its application to a source position searching of the human eye fields

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In order to exploit an inverse problem solver in the magnetostatic fields, a new formulation of the inverse problems is proposed in this paper. As a result, it is shown that all of the inverse problems in magnetostatic fields can be reduced to find the position and magnitude of field source. By combining this new formulation, the finite-element method, and the artificial intelligence technique, we work out a general purpose inverse problem solver in the magnetostatic fields. Intensive tests of this solver suggest that the ability of this solver is depending greatly on the available data base. This solver is now applied to find a source position of the human eye fields. As a result, a plausible source position of the human eye fields can be estimated.

I. INTRODUCTION

With the developments of modern digital computers, numerical methods are widely accepted as an engineering tool in many laboratories. However, in these applications of numerical method, the electromagnetic performance of the devices is of primary concern, because the computational design of electromagnetic devices is essentially reduced to solve an inverse problem for which a methodology has not been well established. The inverse problem is an important problem for device designing as well as the nondestructive testing, particularly medical diagnosis. In the medical inverse problems most of the problems are reduced to find the position and magnitude of a single-current doublet.[1] On the other hand, the engineering inverse problems have several variations, such as the field source position searching and the shape identification of a known or unknown medium.[2-4]

In this paper we propose a new formulation of the inverse problems in the magnetostatic fields.[5] This new formulation makes it possible to work out a general purpose inverse problem solver because all of the inverse problems in magnetostatic fields can be reduced to find the position and magnitude of field source by the formulation. Based on this formulation, we propose a simple inverse problem solver that utilizes the finite elements and AI techniques. Intensive tests of this solver suggest that the ability of this solver is depending greatly on the available data base. This solver is now applied to find a source position searching of the human eye fields. As a result, it is estimated that a source position of the eye magnetic fields is locating mainly at an upper vicinity of the eye.[6,7]

II. THE INVERSE PROBLEMS IN MAGNETOSTATICS

A. Basic equations

Most of the magnetostatic field problems can be reduced to solve a following equation:

$$\nabla\times(1/\mu)\nabla\times A=J_s, \tag{1}$$

where A, J_s, and μ are the vector potential, current density, and permeability. The vector potential A is related to a flux density B by

$$\nabla\times A=B. \tag{2}$$

When we apply a finite-element method to (1), then it is possible to obtain the following system equation:

$$CX=F, \tag{3}$$

where C, X, and F are the coefficient matrix, potential, and input vectors, respectively. Equation (3) is a basic system equation.

B. Formulation of the inverse problems

The inverse problems are roughly classified into two major categories: one is the position and magnitude searchings of the input vector F. The other is the identification of an unknown medium.

At first, let us consider the searching of the unknown input vector F from the field distortions. This input vector F can simply be obtained by substituting a known potential vector X into (3) if all of the potentials at the mesh points are known. However, in most cases, we can obtain only a limited part of an entire potential vector X so that it is not an easy task to find a unique input vector F.

Second, let us consider the identification of an unknown medium assuming that major part of the problem region is occupied by a known medium (e.g., air) and a remaining minor region is occupied by an unknown medium. In this case, (3) can be modified to

$$C_xX=F \tag{4a}$$

or

$$CX=F+(C-C_x)X=F+F_x, \tag{4b}$$

where C_x denotes a coefficient matrix taking into account the unknown medium, C is a coefficient matrix depending on the known medium, F_x is an input vector due to the existence of unknown medium, and F is a known input vector. Obviously, the potential vector X consists of the two components caused by the vectors F and F_x, that is,

$$X=C^{-1}F+C^{-1}F_x=X_k+X_x, \tag{5}$$

so that the identification of the unknown medium is reduced to find the input vector F_x from the following system equation:

$$C(X-X_k)=F_x, \tag{6}$$

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where X_k is a known potential vector computed by $C^{-1}F$.

Thus, the identification of the unknown medium could be reduced to a similar problem of input force vector F searching.[5]

C. Solution of the inverse problems by pattern searching

Let us consider an mth order system equation of (1); then a following equation can be established:

$$C^{-1}F = DF = X, \quad (7)$$

where D is an inverse matrix of C. When we are given the partial potentials x_i ($i = 1$–n, $m > n$) in the entire potential vector X of (7), then a system equation of the inverse problems is written as

$$D'F = X', \quad (8)$$

where D' is a n by m rectangular matrix, and X' is an nth-order vector (whose elements are x_i, $i = 1$–n). In order to determine the input vector F, we compare a pattern due to X' with those of a column matrix d_k ($k = 1$–m) given by

$$d_k = [d_{1,k}, d_{2,k}, \cdot, d_{n,k}]^T, \quad k = 1\text{–}m, \quad (9)$$

in D'. A superscript T in (9) refers to the transpose of matrix. A pattern matching figure γ_k is defined by

$$\gamma_k = \langle X', d_k \rangle / [\|X'\| \cdot \|d_k\|], \quad k = 1\text{–}m. \quad (10)$$

According to the Cauchy–Schwarz inequality in a real vector space, this pattern matching figure γ_k will always be between -1 and 1. If X' and d_k are orthogonal, then γ_k will equal zero. Thus, a maximum absolute value $|\gamma_k|$ identifies a most dominant input source position. After finding the first input source position p ($1 \geqslant p \geqslant m$), this input source pattern is deleted by multiplying the following nth-order square matrix:

$$S_p = \begin{vmatrix} d_{2,p} & -d_{1,p} & 0 & \cdot & 0 & 0 \\ 0 & d_{3,p} & -d_{2,p} & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & d_{n-1,p} & -d_{n-2,p} & 0 \\ 0 & 0 & 0 & \cdot & 1 & 0 \\ 0 & 0 & 0 & \cdot & 0 & 1 \end{vmatrix}, \quad (11)$$

to the left-hand side of (8). This yields

$$D''F = X'', \quad (12)$$

where $D'' = S_p D'$ and $X'' = S_p X'$.

Second, third, and finally, $(n-1)$th patterns can be found in much the same way as (8)–(12). It must be noted that the pattern deleting matrix S_p in (11) makes the 1 to $(n-2)$th elements of the pth column in D', i.e., d_{qp} ($q = 1$–$n-2$), set to be zero; in the second pattern searching, its pattern deleting matrix makes the 1 to $(n-3)$th elements of a corresponding column in the D'' set to be zero; and so on.

After finishing the $(n-1)$th input position searchings, the magnitudes of the input source are calculated by the backward substitutions like a Gaussian elimination technique.[6,8] In other words, our solving method is based on a combination of pattern searching with Gaussian elimination process. We have been given the n points known potentials, but the $n-1$ points input sources were searched by the above process. The remaining potential is utilized to check up the result by defining a following percentage residual r:

$$r = [\|X' - D'F_x\| / \|X'\|] \times 100, \quad (13)$$

where F_x is a solution vector containing the $(n-1)$ obtained inputs and $(m-n+1)$ zero elements.

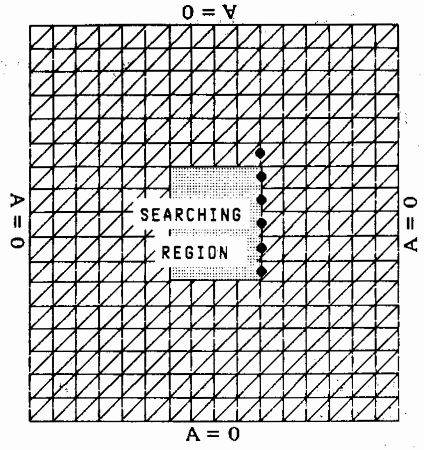

FIG. 1. The boundary condition, mesh system, searching area, and known (or measured) points for a two-dimensional inverse problem. Six known flux densities in the direction of the x axis and 24 possible input points. ● refers to the measured points.

D. An application

At first, the patterns d_k in (9) had been so rearranged by (2) as to use the magnetic flux density B instead of the nodal potentials. The initial tests yielded all of the correct single input positions assuming the six known flux densities and 24 possible input points in Fig. 1. But only about a 30% correct result could be obtained for the two input sources. To remove this deficiency, we provided the additional patterns which were composed of all possible pairs of the column vector (9) to the matrix D' of (8). This database technique

TABLE I. The results of numerical tests.

Number of input points	Included errors (%)	Position	Residual r (%)	Position (result)
1	0	All	0	Correct
2	0	All	0	Correct
2	2	Fixed	2.3793	Correct
2	4	Fixed	4.7628	Correct
2	6	Fixed	7.1631	Correct
2	8	Fixed	9.5336	Correct
2	10	Fixed	4.3038	Correct
2	12	Fixed	16.6690	Partially correct
2	14	Fixed	25.4180	Partially correct
2	16	Fixed	33.2210	Partially correct
2	18	Fixed	24.2410	Partially correct
2	20	Fixed	28.8700	Partially correct
2	22	Fixed	30.5430	Incorrect

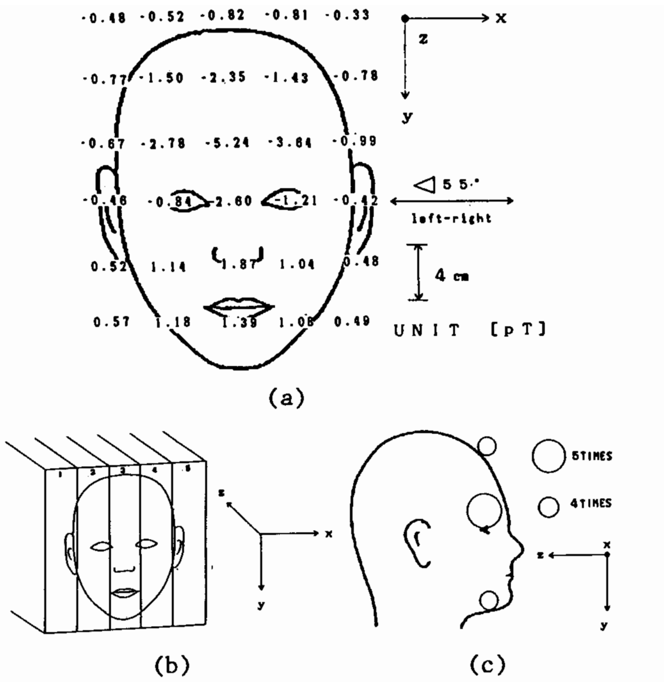

FIG. 2. (a) The measured magnetic flux density distribution in the direction of the z axis when the eyes were moved horizontally over an angle 55° from right to left. This field distribution is caused by a current flowing in the direction of the x axis. (b) Decomposition to the five two-dimensional inverse problems for the position searchings of the x direction source current. (c) The five and four times hit points by the pattern searching and database techniques. O refers to an estimated position of source current.

essentially enhanced a singularity of the system, therefore the program was so modified as to finish searching when an absolute value of the denominator in (10) became a sufficiently small value.

Thus, we succeeded to find all of the correct two input positions as described in Table I. Furthermore, Table I suggests that the correct positions of two input sources may be found by our databased pattern searching technique even if the known potentials include the errors within 10%. Also, it is found that most of the estimated errors by (13) give somewhat larger values than the actually included errors, but it is difficult to use (13) for a decision of unique solution.

Katila and others measured the magnetic fields accompanying the human eye movements.[9] The result is shown in Fig. 2(a). In order to find a source current position in the direction of the x axis, we decomposed this problem region into five two-dimensional problem regions, as shown in Fig. 2(b). Using the mesh system shown in Fig. 1, we carried out the position searchings of a source current for the five distinct two-dimensional problems. The five and four times hit points are shown in Fig. 2(c). The results in Fig. 2(c) reveal that the position of the main source current locates an upper vicinity of the eye. An average current at this point is about $-16\,\mu$A. The average currents of the four times hit points at the up and down sides are about 0.9 and 12 μA, respectively. Therefore, it may be considered that a remaining current $-3\ \mu$A ($\doteqdot -16 + 0.9 + 12$) is flowing over the other paths to satisfy the current continuity. This is a quite plausible result.

III. CONCLUSION

As shown above, we have proposed a new formulation and solution method for inverse problems in magnetostatics. Our new formulation has clarified that the identification of an unknown medium can be reduced to an input source position searching. Also, it has been shown that the pattern searching method utilizing the database technique may become a promising tool for inverse problems. As a practical example, we have applied our method to a source position searching of the human eye magnetic fields. As a result, it has been suggested that the field source of human eye fields is mainly located at an upper vicinity of the eye.

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