The strategic dual image method: An extremely simple procedure for open field problems

Y. Saito, K. Takahashi, and S. Hayano
College of Engineering, Hosei University, 3-7-2 Kajinocho Koganei, Tokyo 184, Japan

Previously we proposed a new method which is based on the strategic dual image forcing an open boundary to close for the finite element solution of open boundary problems. In this paper the relationship between the strategic dual image and traditional electric image methods is clarified. This leads to a method of determining a hypothetical boundary for our strategic dual image method. Several examples demonstrate the versatility of our new method.

I. INTRODUCTION

Because of its essential feature, it is difficult to apply the finite element method to field problems with an open boundary. In order to overcome this difficulty, various means have been devised. These methods are roughly classified into two categories: one is based on the combination of boundary and finite element methods; the other is the infinite and exterior finite element methods.[1–5] In spite of these efforts, it is still required to invent a deterministic method for open field problems because the existing methods require considerable programming effort and computer time compared with those of the conventional finite element method.

Previously, a new method which is based on the strategic dual image forcing an open boundary to close was proposed for the finite element solution of open field problems.[6] This new method is called the strategic dual image method (SDI). Even though our SDI method is capable of any open field problems, a difficulty with its hypothetical boundary determination has been pointed out.

In this paper, the relationship between the SDI and traditional electric image methods is clarified. This leads to a method of determining the hypothetical boundary for the SDI method. Also, it is revealed that our SDI method can be applied to three-dimensional open field problems. Several examples demonstrate the versatility of our SDI method for the open field problems.

II. THE STRATEGIC DUAL IMAGE METHOD

A. Assumptions

At first, the net magnetic field source is assumed to be zero in the problem region. Second, as shown in Fig. 1, it is assumed that the problem region is enclosed by a boundary located at an infinitely long distance from the problem region. This condition means that the boundary conditions at this infinite boundary are

$$B_n = 0, \tag{1}$$

$$H_t = 0, \tag{2}$$

where the flux density B_n and field intensity H_t are, respectively, the normal and tangential components to the infinite boundary as illustrated in Fig. 1. Equations (1) and (2) suggest that the open field is composed of the divergence field B_n and rotational field H_t. Finally, it is assumed that the magnetic field source may be regarded as a rotational field source, i.e., current i, or a divergence field source, i.e., magnetic charge m.

B. Rotational field

Let consider one of the currents i in the problem region. When we impose an image current $-(d/a)i$ at a position shown in Fig. 2(a), then the normal component of flux density B becomes zero at an arbitrary point on the spherical surface enclosing the source current i. This means that the normal component of flux density B to the spherical surface is suppressed by the image current $-(d/a)i$ so that the remaining field is the tangential field intensity H_t to the spherical surface. This field intensity H_t becomes zero at the infinite boundary. The magnitude of image $-(d/a)i$ depends on a position of field source i in the sphere so that the following condition,

$$a \sum_{p=1}^{q} (i_p/r_p) = 0, \tag{3}$$

must be satisfied to reduce the zero net image. In Eq. (3), a is a radius of sphere; $r_p\ (=a^2/d_p)$ is a distance from the center of sphere to the current i_p; and q is a number of sources.

C. Divergence field

Let us consider one of the magnetic charges m in the problem region. When we impose an image $-(d/a)m$ at a position shown in Fig. 2(b), then the tangential component of field intensity H becomes zero at an arbitrary point on the spherical surface enclosing the field source m. This means that the tangential component of field intensity H to the

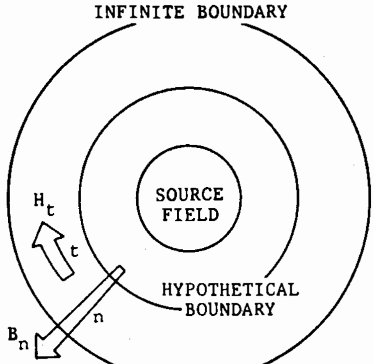

FIG. 1. Modeling of open field problem.

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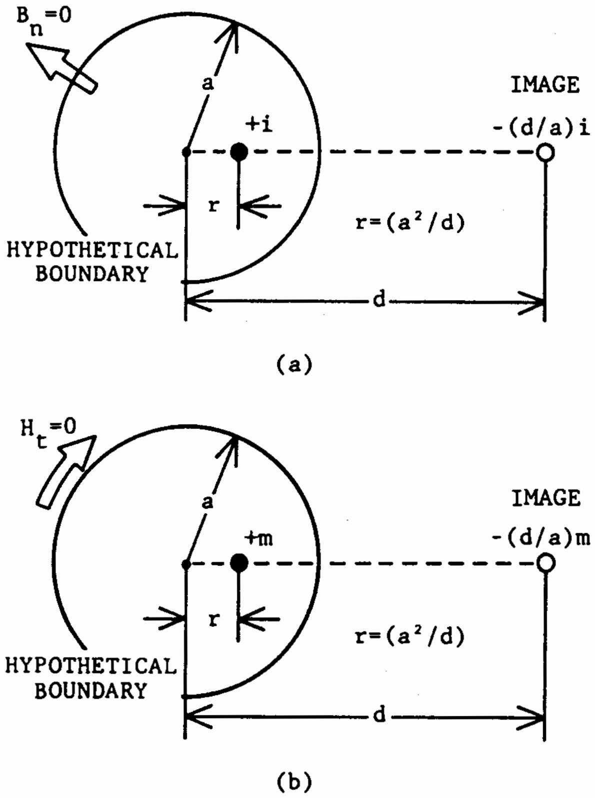

FIG. 2. Strategic dual images. (a) The rotational field source image $-(d/a)i$ and spherical hypothetical boundary. The normal components of flux density at the spherical surface become zero. (b) The divergence field source image $-(d/a)m$ and spherical hypothetical boundary. The tangential components of field intensity at the spherical surface become zero.

spherical surface is suppressed by the image $-(d/a)m$ so that the remaining field in the sphere is the normal component of flux density B_n to the spherical surface. This flux density B_n becomes zero at the infinite boundary. Similar to those of Eq. (3), the following condition,

$$a\sum_{p=1}^{q}(m_p/r_p)=0, \tag{4}$$

must be satisfied to reduce the zero net image.

D. Open field

The field intensity H_t which satisfies condition (2) is obtained by imposing the image of the rotational field source. Also, the flux density B_n which satisfies condition (1) is obtained by imposing the image of the divergence field source. Therefore, the open field can be obtained by a summation of these two fields. It must be noted that an open field thus obtained has two field sources (rotational and divergence field sources) so that the original open field is obtained by dividing the total field by 2, that is:

$$\text{open field} = (1/2)(\text{rotational field} + \text{divergence field}). \tag{5}$$

E. Implementation

If the magnetic field is represented by the curl of vector potential A, then conditions (1) and (2) are respectively satisfied by setting $A=0$ and $\partial A/\partial n=0$ at the spherical surface of the hypothetical boundary, where n denotes a normal direction to the spherical surface of the hypothetical boundary. Furthermore, because of Eq. (3), $A=0$ must be satisfied at the center of the spherical surface.

Similarly, if the magnetic field is represented by the gradient of scalar potential U, then conditions (1) and (2) are respectively satisfied by setting $\partial U/\partial n=0$ and $U=0$ at the spherical surface of the hypothetical boundary. Furthermore, because of Eq. (4), $U=0$ must be satisfied at the center of the spherical surface.

After a governing equation is discretized by the finite element method imposing the zero or symmetrical boundary conditions to the spherical hypothetical boundary, the distributed field source is substantially concentrated at the node points so that conditions (3) and (4) may be easily satisfied for the distributed field source.

When the symmetrical boundary solution vector X_s and the zero boundary solution vector X_z are obtained after solving each of their systems, then the open field solution vector X is obtained by

$$X=(1/2)(X_s+X_z). \tag{6}$$

In Eq. (6), it is required to use the symmetrical and zero boundary solutions. But it is possible to show that our SDI method does not require double computations for symmetrical and zero boundary solutions. This is based on the following fact:

$$X=(1/2)X_s \tag{7}$$

is established at the hypothetical boundary because the other solution vector X_z is always zero.

On the other side, derivative $\partial X/\partial n$ at the hypothetical boundary becomes

$$\frac{\partial X}{\partial n}=\frac{1}{2}\frac{\partial X_z}{\partial n}, \tag{8}$$

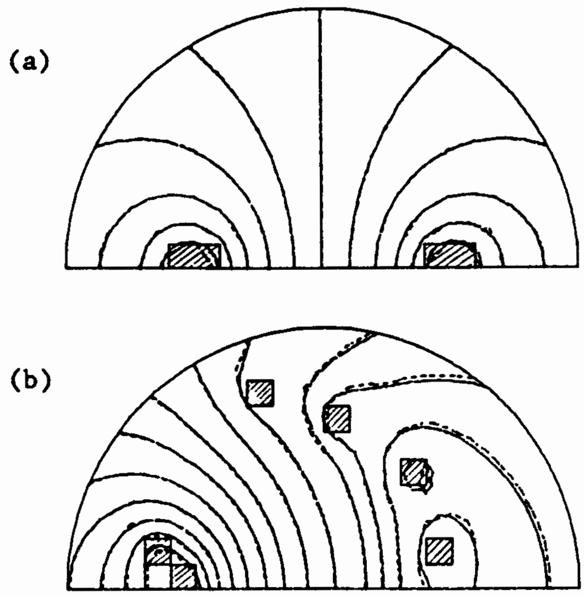

FIG. 3. Two-dimensional magnetostatic fields. (a) Magnetic field distribution of a pair of bifilar conductors. (b) Magnetic field distribution of the solenoidal coils. The solid and dotted lines denote the analytical and computed values, respectively.

because derivative $\partial X_s/\partial n$ of solution X_s is zero. Our SDI method can be implemented by using either boundary condition (7) or (8) and not requiring the double computations.[6]

F. Examples

In this paper, the strategic dual image method has been formulated in three dimensions. However, because of its simplicity, the two-dimensional static magnetic field problems are far more preferable as the concrete examples.

The discretization of examples was carried out by means of first-order triangular finite elements.

Because of a symmetrical property of the problem regions, the computations were carried out on the upper half portion of an entire problem region.

Figure 3(a) shows a magnetic field distribution of a pair of bifilar conductors. A circular hypothetical boundary is well approximated by the triangular elements. Our SDI method yields an excellent result coinciding with those of the analytical method.

Figure 3(b) shows a magnetic field distribution of the solenoidal coils. In spite of a fairly complex field distribution, our new method still yields a good result.

III. CONCLUSION

Our strategic dual image method is an obvious generalization of the traditional electric image method. As a result, it has been elucidated that the position and size of the spherical hypothetical boundary depend on the position of the field source.

In this paper, our new method has been illustrated by the two-dimensional static magnetic field problems. However, our strategic dual image method has been formulated in three dimensions so that this method is obviously applicable to the three-dimensional open field problems.

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