FINITE ELEMENT SOLUTION OF OPEN BOUNDARY MAGNETIC FIELD PROBLEMS

Y.Saito, K.Takahashi and S.Hayano

College of Engineering,Hosei University, Kajino Koganei
Tokyo 184, Japan

Abstract: A simple approximation method of solution is proposed for the open boundary magnetic filed problems. This new method is based on the following assumption: the symmetirical and zero boundary conditions are held at an infinitely long distance from a source point. These conditions are approximately realized on a hypothetical boundary located at a finite distance from a source.Several numerical tests suggests that our new method yields a fairly good approximation of exact solution.

INTRODUCTION

The finite element method has attracted the attention of analysts largely because of its applicabilities to the large number of engineering problems. In fact, extensive theoretical investigations in recent years have made it clear that the finite element method is very general approximation process, and capable of solving most properly defined continuum problems. The finite element method however has certain difficulties. One of the difficulties is manifested when modeling a physical system having open boundary. In order to overcome this difficulty, various means have been devised.These methods may be classified into two types: (1) combination of boundary element and finite element method; (2) the infinite and exterior finite elements approaches.Unfortunately,these existing methods require considerable computational time compared with those of the conventional finite element method [1-4].

In the present paper,a simple approximation method of solution for open boundary magnetic field problems is proposed. This new method is based on the following assumption: the magnetic potential as well as magnetic field intensity become to be zero at an infinitely long distance from a source point. This means that the magnetic potential satisfies the symmetrical and zero boundary conditions at the infinitely long distance point.If we can simultaneously realize these two conditions on a boundary faced to the open region, then this hypothetical boundary may be regarded as being located the infinitely long distance from a source point.In the other words, both of the divergence and rotational fields become to be zero at the infinitely long distance from a source point, so that satisfaction of these conditions leads to a method of solution for the open boundary magnetic field problems.

Establishment of non-divergence and irrotational fields on the hypothetical boundary located at a finite distance from a source point is so difficult that the solution is approximated with a mean of solutions evaluated by imposing the symmetrical and zero boundary conditions on the hypothetical boundary. Thus, our new method is quite simple so that the programming effort and computational time are dramatically reduced. This new method is called the strategic dual image "SDI" method because establishment of symmetrical and zero boundary conditions on the hypothetical boundary is carried out by strategically imposing the dual image of source.

THE STRATEGIC DUAL IMAGE METHOD

Principle

In most of the fields appeared in physical system,the field intensity decreases following to move away from a source point.In addition to decrease of field intensity, the potential may be reduced to be zero, so that both of the field intensity and potential may become to be zero at an infinitely long distance from a source point. This means that the symmetrical and zero boundary conditions may be held at the infinitely long distance from a source point. If it is possible to establish these boundary conditions on a hypothetical boundary located at a finite distance from a source point, then this hypothetical boundary may be regarded as being located at an infinitely long distance. However, it is difficult to establish the symmetrical and zero boundary conditions on the hypothetical boundary so that these boundary conditions are approximately satisfied by taking a mean of the symmetrical and zero boundary solutions. In the other sense, the symmetrical boundary solution gives a upper bounded solution and the zero boundary solution gives a lower bounded solution, therefore the mean of both solutions becomes to a good approximation.

Establishment of the symmetrical and zero boundary solutions is carried out by means of strategic dual image. This is best illustrated by a simple example. Let assume a point charge Q on a free space as shown in Fig. 1(a), then an electric field intensity decreases following to move away from the charge Q. When the dual images +Q and -Q are strategically assumed to be set at x=2L as shown in Figs. 1(b), 1(c), then the positive charge +Q in Fig. 1(b) yields a symmetrical boundary condition at x=L and the other negative charge -Q in Fig.1(c) yields a zero boundary condition at x=L. Since a summation of the charges +Q in Fig.1(b) and -Q in Fig. 1(c) becomes to be zero, then the symmetrical and zero boundary conditions are approximately satisfied at x=L by taking a mean of the symmetrical and zero boundary solutions.

Namely, when the symmetrical and zero boundary solution vectors are evaluated after a governing equation is discretized to a system of equations by the finite elements, then the SDI solution vector X is

$$X = (1/2)[X_s + X_z], \tag{1}$$

where X_s, X_z are respectively the symmetrical and zero boundary solution vectors.

Implementation

To implement our SDI method, it may be required to use the symmetrical and zero boundary solutions.However, it is possible to show that SDI method does not require the double computations for symmetrical and zero boundary solutions. This is based on a following fact: on the hypothetical boundary,

$$X = (1/2)X_s, \tag{2}$$

0018-9464/87/0900-3569$01.00©1987 IEEE

is established because the zero boundary solution vector X_z is always zero.

Let assume a discretized system of equations:

$$A\ X = Y, \tag{3}$$

where A,X,Y are respectively the system matrix,solution vector and input vector; and the symmetrical boundary condition on the hypothetical boundary is assumed. (3) is rewritten as

$$\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \times \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} Y_1 \\ Y_2 \end{vmatrix}, \tag{4}$$

where X_1 is a sub-vector on the inside region; X_2 is a sub-vector on the hypothetical boundary; and the other vectors Y_1,Y_2 and sub-matrices $A_{11},A_{12},A_{21},A_{22}$ are correspondingly defined to the sub-vectors X_1,X_2. From (4), it is possible to obtain the followings:

$$X_1 = A_{11}^{-1}[Y_1 - A_{12}\ X_2], \tag{5}$$

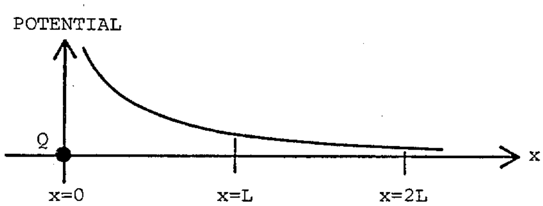
$$X_2 = A_{22}^{-1}[Y_2 - A_{21}\ X_1]. \tag{6}$$

Substituting (5) into (6) yields

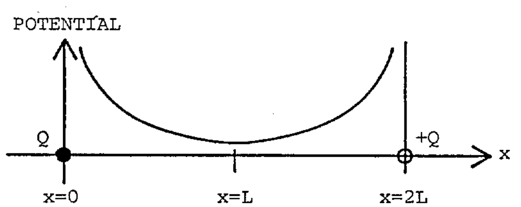
$$X_2 = A_{22}^{-1}[Y_2 - A_{21}A_{11}^{-1}(Y_1 - A_{12}X_2)]. \tag{7}$$

Rearrangement of (7) gives the solution vector X_2 on the hypothetical boundary as

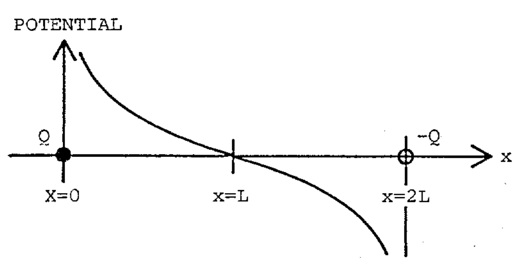
$$X_2 = [A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1}[Y_2 - A_{21}A_{11}^{-1}Y_1]. \tag{8}$$



(a) A source charge Q located at x=0.



(b) A strategic image +Q forcing to construct a symmetrical boundary at x=L.



(c) A strategic image -Q forcing to construct a zero boundary at x=L.

Fig. 1. An illustrative example of the SDI method.

By considering (2),(5) and (8), we can obtain the SDI solution vectors as

$$X_1 = A_{11}^{-1}[Y_1 - A_{12}(1/2)(A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}(Y_2 - A_{21}A_{11}^{-1}Y_1)], \tag{9}$$

$$X_2 = (1/2)[A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1}[Y_2 - A_{21}A_{11}^{-1}Y_1]. \tag{10}$$

On the other side, the derivative $\partial X/\partial x$ on the hypothetical boundary becomes to

$$\partial X/\partial x = (1/2)\partial X_z/\partial x, \tag{11}$$

because the derivative $\partial X_s/\partial x$ of symmetrical boundary solution vector is always zero. By means of (11), the SDI solution vector X can be obtained in much the same way as (2)-(10). Of course, both solution vectors obtained from the conditions (2) and (11) becomes to be identical. Thus, it has been shown that our SDI method can be implemented by using only one boundary condition of (2) or (11) not requiring the double computations.

Numerical Examinations

Various open boundary field problems may be solved by our strategic dual image method. Because of its simplicity, a two dimensional static magnetic field problem is preferable as an initial test. Fig. 2 shows a schematic diagram of the bifiler conductors for which is used as a concrete example. All of the discretizations of this example were carried out by means of a first order triangular finite element method using the right-angled isosceles triangles.

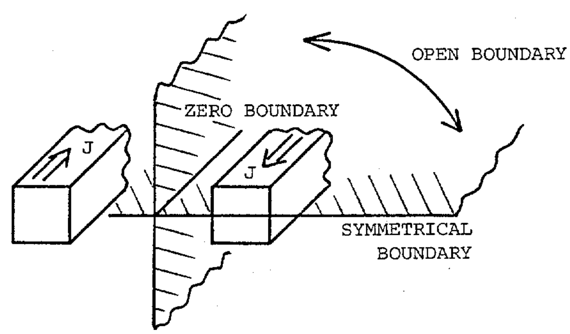
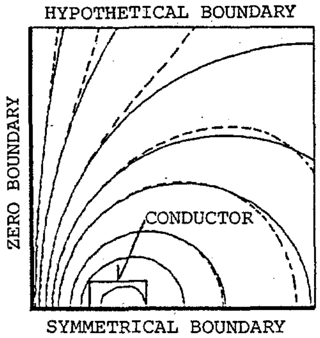


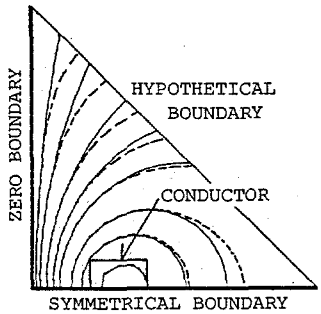
Fig. 2. Schematic diagram of bifiler conductors with square cross-section. J denotes a current density in $[A/m^2]$. A cross-sectional area of conductor is $1x1[m^2]$.

At first, we have evaluated the SDI solutions using the different shapes of hypothetical boundary. Fig. 3 shows the results of computation together with the analytical solutions. By considering the results in Fig. 3, it is found that the SDI solutions greatly depend on the shapes of hypothetical boundary, and the solutions with circular hypothetical boudary centered at a mid point between the bifiler conductors are the excellent results as shown in Fig. 3(c).

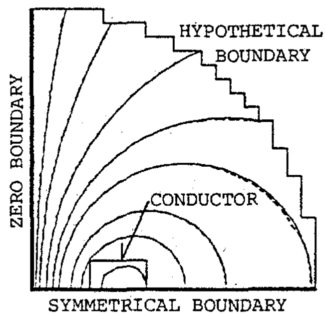
Secondary, we have evaluated the SDI solutions changing the areas of problem region. As a result,it has revealed that the SDI method using circular hypothetical boundary yields the promising results even through the problem region is shrunk to the small regions. Fig. 4 shows an example.



(a) Square hypothetical boundary, where 800 elements were used.



(b) Straight line hypothetical boundary, where 400 elements were used.



(c) Circular hypothetical boundary centered at a mid point between the bifiler conductors,where 640 elements were used.

Fig. 3. SDI solutions with various shapes of the hypothetical boundary, where J=1; the solid lines denote the analytical solutions;and the dotted lines denote the SDI solutions.

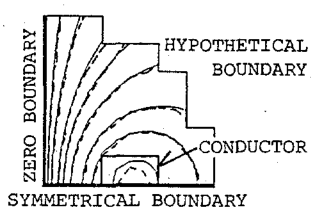


Fig. 4. SDI solution when a problem area was shrunk to a small area, where 60 elements were used. The solid and dotted lines respectively denote the analytical and SDI solutions.

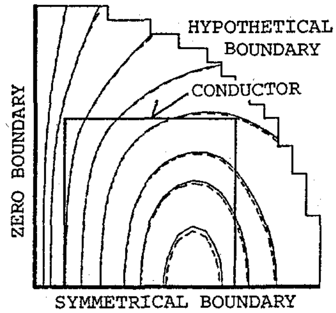


Fig. 5. SDI solution when the big size conductors were used as the bifiler conductors. The cross-sectional area of conductor is 6x3 $[m^2]$. 640 elements were used. The solid and dotted lines respectively denote the analytical and SDI solutions.

Finally, we have evaluated the SDI solutions changing the shapes of conductors. Fig. 5 suggests that the SDI method is still effective means for the different shapes of the bifiler conductors as long as the hypothetical boundary is keeping the circular boundary centered at a mid point between the conductors.

In summarizing the numerical examinations, the circular hypothetical boundaries always provided the good results. This suggests that our SDI method may be closely related with the traditional image method appeared in the text books of electromagnetics. Thus, the relationships between them should be investigated, and this may lead the determination method of position for the circular boundaries. Furthermore, by observing the results in Fig. 4, it is revealed that the numerical solutions are somewhat different from the analyical values. This discrepancy might be caused by the FEM discretization, because an extremely small number of elements was used in the calculation of Fig. 4.

CONCLUSION

As shown above, we have proposed a new approximation method of solutions called the strategic dual image method for the open bondary magnetic field problems. The SDI method has been illustrated by a two-dimensional static magnetic field problem.It is obvious that the SDI method may be applicable not only to the two-dimensional static fields but also to the three-dimensional, time varying and nonlinear fields. The new method is so simple that the programming effort and computer time may be possible to reduce dramatically compared with those of the existing method.

REFERENCES

[1] Meunier, G., et al, "Hybrid finite element boundary element solutions for three-dimensional scaler potential problems," IEEE Transaction on Magnetics Vol. MAG-22, No. 5, Sep. 1986, pp. 1040-1042.

[2] Brebbia,C.A., Editor, Progress in Boundary Element Method, Vol. 1(Pentech Press Ltd., 1981).

[3] Silvester, P.P., et al, "Exterior finite elements for 2-dimensional field problems with open boundaries," Proc. IEE, Vol. 124, No. 12, Dec. 1977, pp. 1267-1270.

[4] Bettess, P., "Infinite elements," Int.J.Num.Meth. Engng., Vol. 11, 1977, pp. 53-64.