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VELOCITY CORRELATION ANALYSIS IN THE NEAR-FIELD OF A TURBULENT JET WITH HELP OF DISCRETE WAVELET TRANSFORM

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ABSTRACT

In this paper, the discrete wavelet transforms using Daubechies orthonormal wavelet bases were applied to the velocity signals in a plane turbulent jet, in order to analyze the eddy motions in the dimension of time and frequency. From the wavelet power spectrum, Daubechies wavelet bases of $N=12$ and 20 had higher resolution that that of $N=4$. The Reynolds stress was decomposed over frequency space by the discrete wavelet transform and negative peaks were observed around $j=8$ and 11 at $y/b_0=1.0$ and around $j=7$ on the centerline. It is called *Reynolds stress reversals* or inverse transfer of energy in the frequency space. Physically, this behavior signified that fluctuation energy was transferred from these frequency components to lower frequency components, and resulted in a decrease of turbulent energy. From the distribution of discrete wavelet coefficients the larger scales, $j=6$, 7 and 8, showed large peaks. These scales were dynamically quite important and indicated dominant eddies.

INTRODUCTION

The plane jet is a commonly used model for study of two-dimensional turbulent flows or eddy structures. Since Crow and Champagne (1971) first studied the coherent structure of a turbulent jet, the physics of the plane turbulent jet has been widely investigated for several decade years (Gutmak, 1976; Moum et al., 1979; Gervantes et al., 1981; Goldshmidt et al., 1981; Krothapalli et al., 1981; Antonia et al., 1983; Oler et al., 1984; Hsiao et al., 1994). It has become well-known fact that the large-scale eddy motion of the plane turbulent jet exhibits a symmetric, periodic and apparent flapping motion in similarity region, and the evolution and interaction of large-scale organized structures play an important role in the turbulent jet spreading and momentum transfer. The conventional statistical methods, such as space-time correlation functions, spectra, coherent functions, conditional sampling methods and visualization techniques are well-established usual techniques for gaining information regarding the nature of turbulent structure or eddy motion. However, the structure of turbulence or eddy motion is unsteady, and is characterized by the existence of multiple spatial scales. One of conventional methods to study the turbulent structure by experimental investigations is to analyze the time-mean characteristics of turbulent flows by quantitative measurements of the flow characteristics, such as velocity, pressure, temperature and so on. But some important informations on the time domain were lost owing to the non-local nature of the Fourier analysis. The visualization of organized motions in shear layers showed that the conditional sampling measurement had been hiding very important features of turbulence (Laufer, 1975).

In recent 15 years, there has been growing interest in the wavelet analysis of signals, which can combine time-space and frequency-space analyses to produce a potentially more revealing picture of time-frequency localization of signals. As a tool for analysis of multiscale signals, the wavelet transform was originated in geophysics in early 1980's for the analysis of seismic signal. Now, wavelet analysis has been formalized into a rigorous mathematical framework and has been applied to numerous diverse areas, such as mathematics, physics, turbulence, signal processing, image processing, numerical analysis, nonlinear dynamics, fractal and multifractal analysis.

However, the wavelet analysis is relatively new to the field of turbulence, having mostly been developed in the present decade, and has not yet been applied to their full potential in this area of fluid mechanics. Until now, numerous papers on this topic have been published, and these research projects can be divided into two groups: (1) the turbulent or eddy structure analysis from the wavelet transforms of experimental and numerical data; and (2) the development of turbulence modeling and numerical methods based on wavelets. In this paper we only discuss the application of the wavelet analysis to the experimental study of jet.

In the limited open literature available, Everson et al. (1990) analyzed two-dimensional dye concentration data from a turbulent jet using the wavelet transform, and revealed the nature and self-similarity of the inner structure of the jet. Lewalle et al. (1994) applied the wavelet transform to velocity signals in the inner mixing layer of a coaxial jet and analyzed the dominance of non-periodic vortices at a given time scale. Li and Nozaki (1995) displayed very different scale eddies, the breakdown of a large eddy and the successive branching of a large eddy structure in a plane turbulent jet by analyzing the velocity signals at various positions with the wavelet transform. A wavelet decomposition of fluctuating velocities in a planar jet was used to the Reynolds stress in scale space by Gordeyev and Thomas (1995). Walker et al. (1995) investigated the multiple acoustic modes and the shear layer instability waves of the jet by wavelet transform. Using wavelet decomposition the unsteady aspects of the transition of a jet shear layer were studied by Gordeyev et al. (1995). In order to reveal the structure of eddy motion and coherent structure in a plane turbulent jet in scale and time delay, Li (1997) proposed a wavelet auto-correlation and wavelet cross-correlation analysis based on the wavelet transform. Besides these application studies, several new tools and diagnostics based on the wavelet transform, such as wavelet intermittency, wavelet Reynolds number, wavelet spectrum, wavelet cross spectrum and wavelet correlation function were developed. They offered the potentials extracting new information from various flow fields. In order to extract new information of turbulent flows, which is lost if conventional statistical methods are used, it is necessary to develop new analytical techniques.

Until now, most of researches on the turbulent structure were carried out by using the continuous wavelet transform. In this paper, the discrete wavelet transforms are applied to the velocity signal of a plane turbulent jet, in order to analyze the eddy motion in the dimension of time and frequency.

DISCRETE WAVELET TRANSFORM

The wavelet transform can either be continuous or discrete, and yields elegant decompositions of turbulent flows. The continuous wavelet transform offers a continuous and redundant unfolding in terms of space and scale and thus can track coherent structures. The discrete wavelet transform allows an orthogonal projection on a minimal number of independent modes. Such analysis is known as a multiresolution representation and might be used to compute or model turbulent flow dynamics. Like the fast Fourier transform, the discrete wavelet transform is a fast, linear operation that operates on data whose length is an integer power of two, and is invertible and in fact orthogonal inverse transform. The details regarding the discrete wavelet transform can be found in many references. Here, a brief review of the discrete wavelet transform is introduced from the view of matrix.

The discrete wavelet transform is a transformation of information from a fine scale to a coarser scale by extracting information that describes the fine scale variability (the detail coefficients or wavelet coefficients) and the coarser scale smoothness (the smooth coefficients or mother-function coefficients) according to:

$$\{S_j\}=[H]\{S_{j-1}\} \quad ; \quad \{D_j\}=[G]\{S_{j-1}\} \qquad (1)$$

where S represents mother-function coefficients, D represents wavelet coefficients, j is the wavelet level, and H and G are the convolution matrices based on the wavelet basis function. High values of j signify finer scales of information. The complete wavelet transform is a process that recursively applied Equation (1) from the finest to the coarsest wavelet level (scale). This describes a scale by scale extraction of the variability information at each scale. The mother-function coefficients generated at each scale are used for the extraction in the next coarser scale.

The inverse discrete wavelet transform is similarly implemented via a recursive recombination of the smooth and detail information from the coarsest to finest wavelet level (scale):

$$\{S_{j-1}\}=[H]^T\{S_j\}+[G]^T\{D_j\} \qquad (2)$$

where H^T and G^T indicate the transpose of H and G matrices, respectively.

The matrices H and G are created from the coefficients of the basis functions, and represent the convolution of the basis function with the data.

Many different orthonormal wavelet basis functions, such as the Harr basis, Daubechies basis, Meyer basis, Spline basis and Coiflets basis, can be used in the discrete wavelet transform. Different wavelet basis functions will preferentially move, between scales, different characteristics of the target data sets. Such as, use of The Harr basis function may emphasize discontinuous in the target data sets, and the Daubechies family may emphasize the smoothness of the analyzed data. In this paper, we use the Daubechies family as the wavelet basis function. The simplest (and most localized) member often called DAUB4, which has only four coefficients, c_0, c_1, c_2 and c_3, is given, by the following matrix.

$$\begin{bmatrix} c_0 & c_1 & c_2 & c_3 & & & & & & \\ c_3 & -c_2 & c_1 & -c_0 & & & & & & \\ & & c_0 & c_1 & c_2 & c_3 & & & & \\ & & c_3 & -c_2 & c_1 & -c_0 & & & & \\ & & & & \cdot & & & & & \\ & & & & & \cdot & & & & \\ & & & & & & \cdot & & & \\ & & & & & & c_0 & c_1 & c_2 & c_3 \\ & & & & & & c_3 & -c_2 & c_1 & -c_0 \\ c_2 & c_3 & & & & & & & c_0 & c_1 \\ c_1 & -c_0 & & & & & & & c_3 & -c_2 \end{bmatrix} \tag{3}$$

where blank entries signify zeroes. The action of the matrix, overall, is thus to perform two related convolutions, then to decimate each of them by half and interleave the remaining halves. It is useful to think of the coefficient c_0, c_1, c_2 and c_3 as being a smoothing filter, call it matrix H, something like a moving average of four points. Then because of the minus signs, coefficient c_3, $-c_2$, c_1 and $-c_0$, call it matrix G, is not a smoothing filter.

Just as, in the discrete Fourier transform, the sequence length $N=2^J$ of the signal being analyzed determines how many separate frequencies can be represented, in the discrete wavelet transform the sequence length determines how many wavelet levels there are. For wavelet level j, the frequency range is centered at $2^{J-j}/\mathit{time\,length}$ when using the Daubechies family (Newland, 1993).

EXPERIMENTAL APPARATUS

A definition sketch of a plane jet is shown in Fig.1, where x is the streamwise coordinate and y is the lateral coordinate. The jet is generated by a blower-type wind tunnel having flow-straightening elements, screens, and settling length; a 24:1 contraction leading to a $300mm$x$20mm$ nozzle (the nozzle width $2b_0$ is $20mm$). For all measurements reported herein the Reynolds number Re, using the nozzle width $2b_0$ as the characteristic length, is fixed at Re=5000 (exit velocity U_0 is $4.8m/s$). At this speed, the exist turbulence level measured on the center of the nozzle is about 1.4 percent. u and v are the fluctuating velocity components in x and y direction, respectively, and the velocity components of u, v are measured simultaneously using an X type hot wire probe located in the (x, y)-plane. The recording frequency is $10kHz$ and the recording length is 6 seconds.

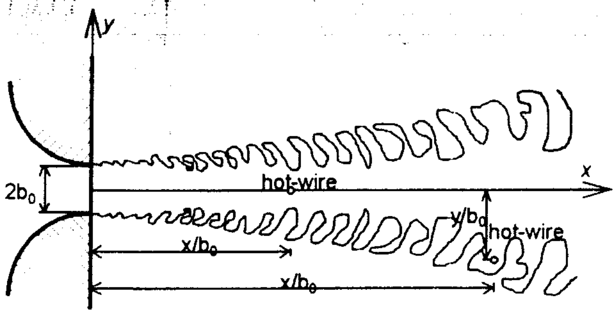

Fig.1 Sketch of the experimental configuration

RESULTS AND DISCUSSIONS

To examine the effect of different wavelet basis functions, the wavelet power spectrum on the jet centerline at x/b_0=4 was computed with Daubechies orthonormal wavelet bases of N=4, 12 and 20, and their results were shown in Fig.2. It is observed that a large peak appeared at wavelet level 8 for three Daubechies orthonormal wavelet bases, which corresponded to the frequency range of $39Hz$ approximately. The results of Daubechies bases with N=12 and 20 had almost same distribution, and indicated same localization in frequency space. However, the peak of Daubechies N=4 was smaller at j=8 and was larger at j=11 than that of N=12 and 20. It implied that Daubechies bases with N=12 and 20 has higher

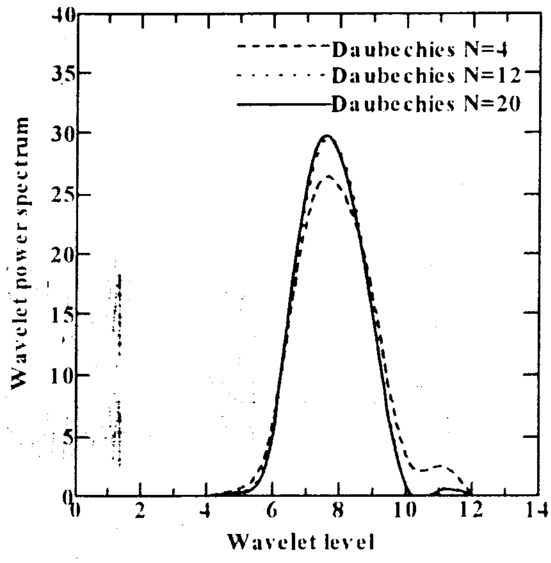

Fig.2 Wavelet power spectrum on the jet centerline at x/b_0=4

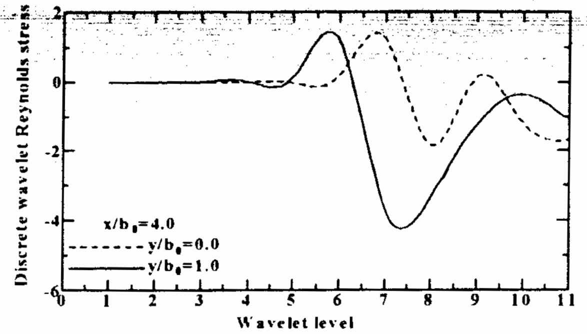

Fig.3 Discrete wavelet Reynolds stress at x/b_0=4

resolution than that of N=4. In this study, we use only the Daubechies bases of N=20 to analyzing the experimental fluctuating velocities.

It is well known that positive turbulent shear stress induces the positive production, while negative turbulent shear stress exhibit the negative production, due to the existence of the negative mean velocity gradient in a jet shear layer region. Until now, the phenomenon of negative production existed in physical space has only been detected in shear layer of excited jet, but distribution of negative production in both frequency space have not been clear yet in a plane jet. In this paper, we employ the discrete wavelet Reynolds stress to decompose the Reynolds stress over frequency space and to analyze the near field characteristics of a plane jet. Figure 3 showed the distribution of the wavelet Reynolds stress at x/b_0=4 and y/b_0=0.0 and 1.0. It was apparent that positive peaks exited around j=6 and 7 in the shear layer (y/b_0=1.0) and on the centerline, respectively, which indicated the maximum turbulent energy and corresponded to the frequency of 156*Hz* and 78*Hz* approximately. Negative peaks are observed around j=8 and 11 at y/b_0=1.0 and around j=7 on the centerline. It is called *Reynolds stress reversals* or inverse transfer of energy. Physically, this behavior signified that fluctuation energy was transferred from these frequency components to lower frequency components at in the shear layer, and resulted in a decrease of turbulent energy. Since sums over all fluctuating components were carried out in the conventional turbulent shear stress, negative values could be balanced instead by local positive values. Hence, it is difficulty to find the information of negative production with the conventional methods. The magnitudes of negative peak in the shear layer were lager than that on the centerline. The interaction between positive and negative production in frequency space leaded to the eddy formation and the merging process in the shear layer, and dominates the jet spreading in the developing region.

In order to reveal the near field flow structure of a jet in both scale and physical spaces, components of fluctuation velocities at x/b_0=4 and y/b_0=1.0 were decomposed using

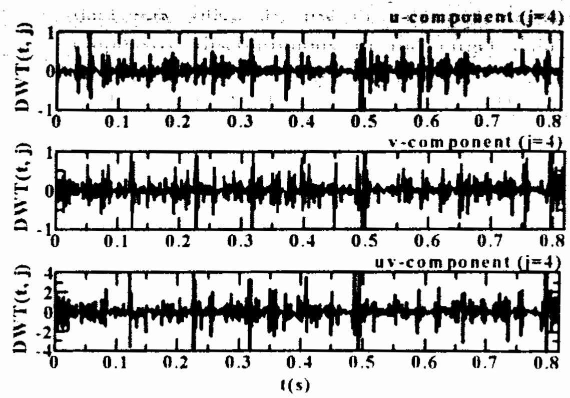

Fig.4 (a) Discrete wavelet decomposition of fluctuation velocities with j=4 at x/b_0=4 and y/b_0=1.0

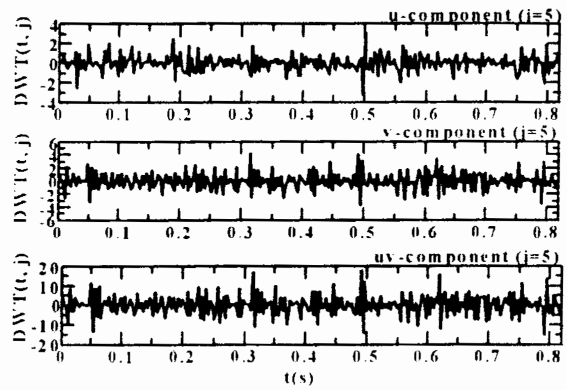

Fig.4 (b) Discrete wavelet decomposition of fluctuation velocities with j=5 at x/b_0=4 and y/b_0=1.0

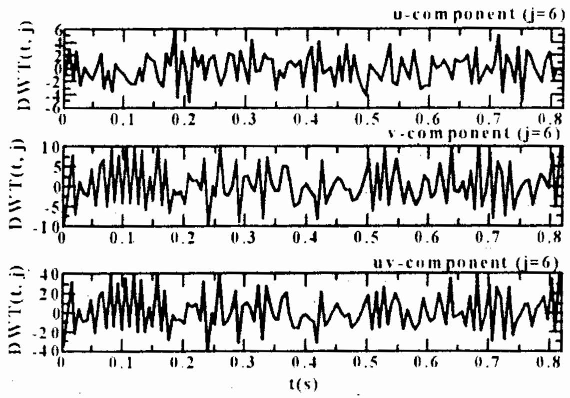

Fig.4 (c) Discrete wavelet decomposition of fluctuation velocities with j=6 at x/b_0=4 and y/b_0=1.0

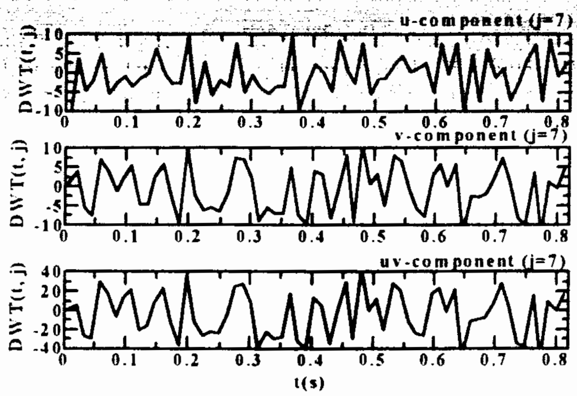

Fig.4 (d) Discrete wavelet decomposition of fluctuation velocities with j=7 at x/b_0=4 and y/b_0=1.0

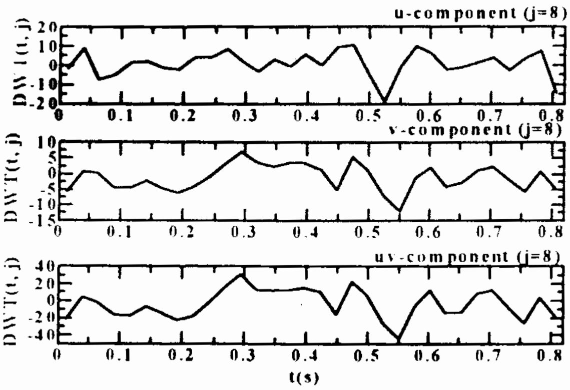

Fig.4 (e) Discrete wavelet decomposition of fluctuation velocities with j=8 at x/b_0=4 and y/b_0=1.0

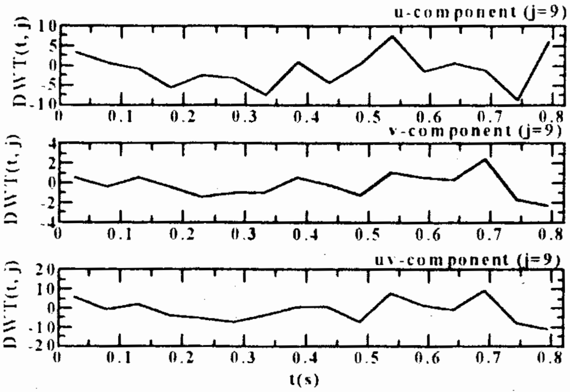

Fig.4 (f) Discrete wavelet decomposition of fluctuation velocities with j=9 at x/b_0=4 and y/b_0=1.0

Daubechies' orthonormal wavelet bases and were shown in Fig.4. Graphics represented the discrete wavelet transform of u-component, v-component and uv fluctuation velocity, respectively. These wavelet levels ranged from 4 to 9 and corresponded to the spectrum frequency range between 19.5Hz and 625Hz approximately.

The experimental results had shown that in the free shear layer the positive peaks in u and v correspond to the passing of eddies, and negative peaks in u and v correspond to the interval between eddies and the entrainment processes, respectively. Therefore, the positive and negative peaks in discrete wavelet transform coefficients of u-component, v-component can describe the process of eddies passing and entrainment processes, respectively.

For the wavelet level j=4, as shown in Fig.4 (a), large peaks related to passage of eddies and entrainment processes with frequency of 625Hz. At j=5 (Fig.4 (b)), a large peak was observed around t=0.5s. When increasing the wavelet level, the time series for scales, j=6, 7 and 8 (Figs.4 (c), (d) and (e)), which corresponded to frequencies of 156Hz, 78Hz and 39Hz, showed large peaks. These scales were dynamically quite important and indicated dominant eddies. From Fig.4 (e), a large negative peak can be observed around t=0.55s in v and uv wavelet coefficients. It indicated a strong entrainment process and local negative contribution to Reynolds stress with frequency of 39Hz in the shear layer, respectively. At j=9 (Fig.4 (f)), magnitudes of wavelet coefficients decreased overall.

CONCLUSIONS

(1) From the wavelet power spectrum, Daubechies wavelet bases of N=12 and 20 has higher resolution that that of N=4.

(2) The Reynolds stress is decomposed over frequency space by discrete wavelet transform and negative peaks were observed around wavelet levels j=8 and 11 at y/b_0=1.0 and around j=7 on the centerline. It is called *Reynolds stress reversals* or inverse transfer of energy. Physically, this behavior signified that fluctuation energy was transferred from these two frequency components to lower frequency components, and resulted in a decrease of turbulent energy.

(3) The time series of discrete wavelet coefficients for the scales, j=6, 7 and 8, showed large peaks. These scales were dynamically quite important and indicated dominant eddies.

REFERENCES

Antonia, R. A., Browne, L. W., Rajagopalan, S. and Chambers, A. J., 1983, "On the Organized Motion of a Turbulent Plane Jet", J. Fluid Mech., Vol.134, pp.49-66.

Crow S.C., and Champagne F. H., 1971, "Orderly Structure in Jet Turbulence", J. Fluid Mech., Vol.48, 547-591.

Everson, R. and Sirovich, L., 1990, "Wavelet Analysis of the Turbulent Jet", Phys. Lett., Vol.145, No.6, pp.314-322.

Gervantes, de Gortari, J. G., and Goldschmidt, V. W., 1981, "The Apparent Flapping Motion of a Turbulent Plane Jet - Further Experimental Results", ASME Journal of Fluids Engineering, Vol.103, No.1, pp.119-126.

Goldschmidt, V.W. and Bradshaw, P., 1973, "Flapping of a Plane Jet", Physics of Fluids, Vol.16, pp.354-355.

Gordeyev, S. V. and Thomas F. O., 1995, "Measurement of Reynolds Stress Reversal in a Planar Jet by Means of a Wavelet Decomposition", Turbulent Flows ASME, FED-Vol.208, pp.49-54.

Gordeyev, S. V., Thomas F. O., and Chu, H. C., 1995, "Experimental Investigation of Unsteady Jet Shear Layer Dynamics Using a Wavelet Decomposition", Unsteady Flows ASME, FED-Vol.216, pp.167-172.

Gutmark, E., 1976, "The Planar Turbulent Jet, J. Fluid Mech., Vol.73, Part 3, pp.465-495.

Hsiao, F. B. and Huang, J. M., 1994, "On the Dynamics of Flow Structure Development in an Excited Plane Jet", ASME Journal of Fluids Engineering, Vol.116, No.4, pp.714-720.

Krothapalli, A., Baganoff, D., and Karamcheti, K., 1981, "On the Mixing of a Rectangular Jet", J. Fluid Mech., Vol.107, pp.201-220.

Laufer, J., 1975, "New Trends in Experimental Turbulence Research", Ann. Rev. Fluid Mech., Vol.7, pp.307-320.

Lewalle, J., Petagna P. and Buresti G., 1994, "Wavelet Statistics of the Near-Field Structure of a Coaxial Jet", AIAA Paper 94-2323.

Li, H. and Nozaki, T., 1995, "Wavelet Analysis for the Plane Turbulent Jet (Analysis of Large Eddy Structure)", JSME International Journal, Fluids and Thermal Engineering, Vol.38, No.4, pp.525-531.

Li, H. and Nozaki, T., 1997, "Application of Wavelet Cross-Correlation Analysis to a Turbulent Plane Jet", JSME International Journal, Fluids and Thermal Engineering, Vol.40, No.1, pp.58-66.

Li, H., 1997, "Wavelet Auto-Correlation Analysis Applied to Eddy Structure Identification of Free Turbulent Shear Flow", JSME International Journal, Fluids and Thermal Engineering, Vol.40, No.4, pp.567-576.

Moum, J. N., Kawall, J. G., and Keffer, J. F., 1979, "Structure Features of the Plane Turbulent Jet", Physics Fluids, Vol.22, No.7, pp.1240-1249.

Newland, D. E., 1993, "An Introduction to Random Vibrations, Spectral & Wavelet Analysis", Longman.

Oler, J. W., and Goldschmidt, V. W., 1984, "Coherent Structures in the Similarity Region of Two-Dimensional Turbulent Jets", ASME Journal of Fluids Engineering, Vol.106, No.2, pp.187-192.

Walker, S. H., Gordeyev, S. V., and Thomas F. O., 1995, "A Wavelet Transform Analysis Applied to Unsteady Jet Screech Resonance", High Speed Jet Flows ASME, FED-Vol.214, pp.103-108.