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A REPRESENTATION OF MAGNETIC HYSTERESIS BY FOURIER SERIES

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The magnetization characteristics are separated into the saturation and hysteretic properties by the Fourier series while the induction is sinusoidally varying with time. These two properties yield the parameters of a Chua type model, which is closely related with the Preisach type models.

1. Introduction

With the development of modern computers, numerical methods became available to calculate the magnetic fields of electromagnetic devices. In order to calculate accurately the fields, it is essential to work out a model which behaves just look like a true magnetic material. Even though a Chua type model is capable of any magnetization properties, a difficulty of its parameter determination has been pointed out.

In the present paper, a couple of parameters of a Chua type model is determined by means of a Fourier series. Also, it is shown that the saturation property is a function of the induction, and the hysteretic property is a function of the time derivative of induction. Furthermore, a Steinmetz type formula for the power loss in the materials is derived. Our formula suggests that the power loss is proportional to the square of peak value of induction within a certain range as long as the correct sinusoidal wave form of the induction is held.

2. Fourier analysis of magnetization characteristics

Fourier analysis is effectively used to clarify the harmonic content of the periodic waves. However, in the present paper, Fourier analysis is available to separate the periodic wave into the odd and even components.

When the induction B is sinusoidally varying with time t, then the associated field intensity H becomes to a nonsinusoidal periodic wave. Fig. 1a shows a B–H loop, and fig. 1b shows the wave forms of B and H. By means of a Fourier series, the field intensity H in fig. 1b is expanded into the sine and cosine series, that is

$$H = \sum_{n=1}^{\infty} H_{sn} \sin(n\omega t) + \sum_{n=1}^{\infty} H_{cn} \cos(n\omega t), \tag{1}$$

where ω denotes the angular velocity of B. Let T be the period of B, then $\omega = (2\pi/T)$, and the Fourier coefficient H_{sn}, H_{cn} for nth harmonics are given by

$$H_{sn} = \frac{2}{T}\int_0^T H \sin(n\omega t)\mathrm{d}t, \tag{2}$$

$$H_{cn} = \frac{2}{T}\int_0^T H \cos(n\omega t)\mathrm{d}t. \tag{3}$$

The odd and even components of H are, respectively, obtained by

$$H_o = \sum_{n=1}^{\infty} H_{sn} \sin(n\omega t), \tag{4}$$

$$H_e = \sum_{n=1}^{\infty} H_{cn} \cos(n\omega t). \tag{5}$$

As shown in figs. 1c and d, the odd component H_o and even component H_e, are, respectively, in phase with the induction B and the time derivative of induction $\mathrm{d}B/\mathrm{d}t$. Thereby, a combination of H_o with B yields one of the saturation curves. Also, a combination of H_e with $\mathrm{d}B/\mathrm{d}t$ yields a curve which represents the hysterestic property, because $H_e(\mathrm{d}B/\mathrm{d}t)$ provides the power loss per unit volume. Figs. 2a and b show the B–H_o and $\mathrm{d}B/\mathrm{d}t$–H_e curves, respectively. In the other words, by

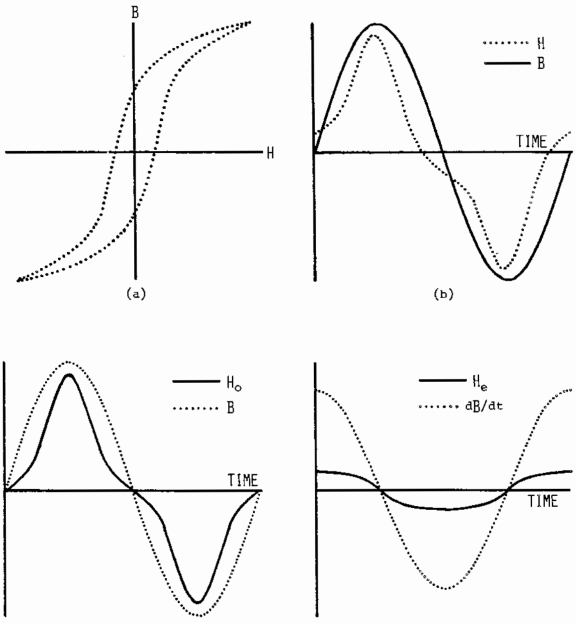

Fig. 1. (a) $B-H$ loop; (b) time variations of B and H; (c) B and odd component H_o of H; and (d) $\mathrm{d}B/\mathrm{d}t$ and even component H_e of H.

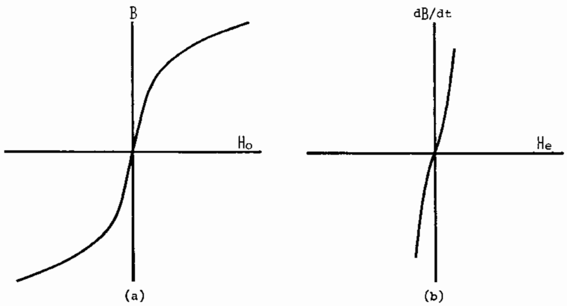

Fig. 2. (a) $B-H_o$ curve which represents the saturation property; and (b) $dB/dt-H_e$ curve which represents the hysteretic property.

considering the relations of figs. 2a and b, it is possible to write the field intensity H as

$$H = H_o + H_e = f_s(B) + f_h(dB/dt), \quad (6)$$

where $f_s(B)$ and $f_h(dB/dt)$ are denoting the single valued functions of B and dB/dt, respectively. This means that the saturation property is a function of the induction B only, and the hysterestic property is a function of the time derivative of induction dB/dt only.

By defining the permeability μ and hysteresis coeficient s as

$$\mu = B/H_o, \quad (7)$$

$$s = (dB/dt)/H_e, \quad (8)$$

the relation (6) is expressed by

$$H = (1/\mu)B + (1/s)dB/dt. \quad (9)$$

The expression (9) is known as a Saito's formula, and it has been reported that the hysteresis coefficient s is closely related with the Preisach's distribution function Ψ [1–3].

Another important magnetization characteristic is the power loss in the materials. We derive here a Steinmetz type formula from a Chua type model (9) under the constraint of the correct sinusoidal wave form of induction.

Since the voltage per unit area is the time derivative of induction dB/dt and the current per unit length is the field intensity H, the instantaneous power P_i per unit volume is given by

$$P_i = H(dB/dt). \quad (10)$$

We are enforcing the sinusoidally time varying induction B with angular velocity ω, so the mean power P_a of instantaneous power P_i is calculated by

$$P_a = (\omega/2)H_{c1}B_m, \quad (11)$$

where H_{c1} and B_m are the Fourier coefficient of 1st order even field in the relation (3) and the peak value of induction B, respectively. Let s_1 be the hysteresis coefficient for the fundamental wave defined as

$$s_1 = \omega(B_m/H_{c1}), \quad (12)$$

then the average power P_a can be expressed by

$$P_a = (1/2s_1)(\omega B_m)^2. \quad (13)$$

Expression (13) is a typical Steinmetz type formula [4] for the hysteresis loss. However, when the induction B and field intensity H in figs. 1a and b are including the eddy current effects, then expression (13) exhibits the losses due to the hysteresis as well as eddy currents. The expression (13) suggests that the power loss P_a must be proportional to the square of B_m and ω as long as s_1 takes a constant value and the correct sinusoidal wave form of the induction B is held.

Although the correct experimental conditions have not been established, the experimental data by Prusty [5] have been fairly followed by formula (13) in our examination.

3. Conclusion

As shown above, an alternative formulation of a Chua type model has been carried out by means of a Fourier series. As a result, it has been clarified that the saturation property depends on the induction, on the other side, the hysteretic property depends on the time derivative of induction. Furthermore, it has been suggested that the power loss must be proportional to the square of peak value of induction within a certain range as long as the correct sinusoidal wave form of the induction is held.

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