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Ferroresonance circuit exhibiting chaotic phenomenon: Rule extraction from nonlinear systems

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Abstract. This paper investigates a rule of nonlinear systems exhibiting chaotic phenomena. A typical ferroresonant circuit is studied by the Chua-type magnetization model representing magnetodynamics. The nonlinear calculation with state variable equation is carried out to clarify the ferroresonant phenomenon. The characteristic values of the state transition matrix are also calculated in each of iteration steps. It reveals that they have no hysteretic properties even though chaotic behavior is exhibiting.

1. Introduction

Many kinds of prediction on the physical systems require carrying out nonlinear calculations. To clarify the rule of such systems is of importance to investigate any systems. In ferromagnetism, we have to take into account magnetic hysteresis, saturation, aftereffects, frequency dependence, etc. With the Chua-type magnetization model a ferroresonant circuit is studied to clarify the rule of nonlinear systems exhibiting a chaotic property. Calculating the characteristic values based on state variable representation reveals that it is possible to obtain no hysteretic properties even if the system exhibits chaotic behavior.

2. Ferroresonant circuit exhibiting chaotic behavior

2.1. Chua-type magnetization model and its parameters

The Chua-type magnetization model has solved various phenomena including ferromagnetic materials excepting for anisotropic materials, resulting in the dynamic constitutive relation between magnetic field H [A/m] and flux density B [T] [1–3]

$$H + \frac{\mu_r}{s}\frac{\partial H}{\partial t} = \frac{1}{\mu}B + \frac{1}{s}\frac{\partial B}{\partial t}, \tag{1}$$

where μ, μ_r, and s are the parameters for Chua-type magnetization model, permeability [H/m], reversible permeablity [H/m], and hysteresis parameter $[\Omega/\mathrm{m}]$, respectively. Figure 1 shows typical parameters of

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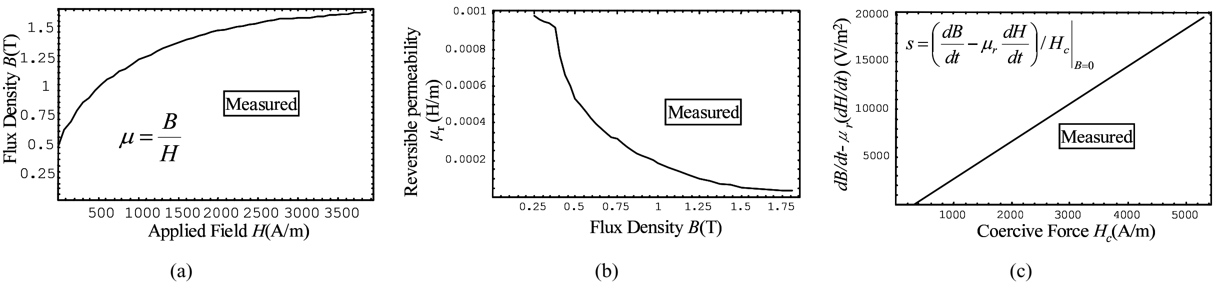


Fig. 1. Parameters of Chua-type magnetization model (Measured: soft iron). (a)Permeability μ. (b)Reversible permeability μ_r. (c)Hysteresis parameter s.

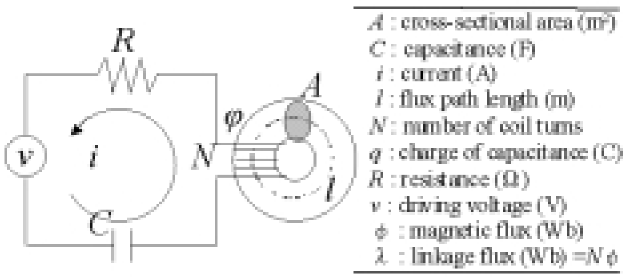


Fig. 2. Series ferroresonant circuit.

the Chua-type model for soft iron. Since we have to represent phenomena exhibiting hysteresis, then we never use any parameters affected by the past magnetization histories. Namely, the parameters should be obtained during ideal magnetization curves are measured. Note that these parameters are also unique even though frequency of the applied field is changed because of the ideal magnetization curve [4].

2.2. *Ferroresonant phenomenon*

Consider a series ferroresonant circuit shown in Fig. 2. At first, the line integral of Eq. (1) along with flux path l yields a relation between the current i and linkage flux λ:

$$Ni + \frac{\mu_r}{s} N \frac{di}{dt} = \frac{l}{\mu A N}\lambda + \frac{l}{sAN}\frac{d\lambda}{dt}. \tag{2}$$

Moreover, the relation between the driving voltage source v and current i is given by

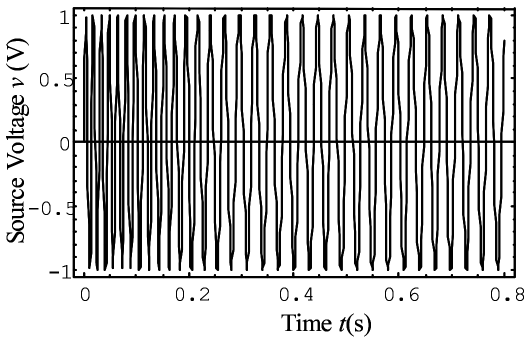
$$i = \frac{1}{R}\left(v - \frac{1}{C}q - \frac{d\lambda}{dt}\right). \tag{3}$$

Second, substituting Eq. (3) into Eq. (2) yields the state equations for the circuit.

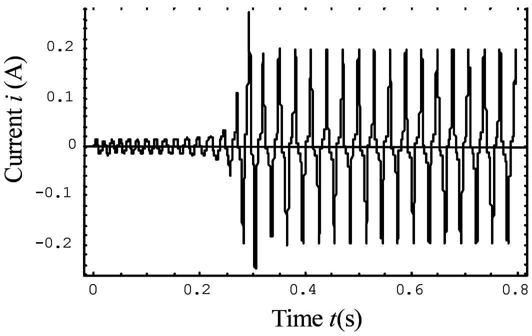
$$\begin{cases} \frac{\mu_r}{s}\frac{d^2\lambda}{dt^2} + \left(1 + \frac{lR}{sAN^2} - \frac{\mu_r}{sRC}\right)\frac{d\lambda}{dt} + \frac{lR}{\mu AN^2}\lambda + \frac{1}{C}\left(1 - \frac{\mu_r}{sRC}\right)q = \frac{\mu_r}{s}\frac{dv}{dt} + \left(1 - \frac{\mu_r}{sRC}\right)v \\ \frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\lambda}{dt} + \frac{q}{C} - v\right) \end{cases}. \tag{4}$$

Table 1
Constants for analysis

μ : permeability [H/m]	Fig. 1(a)
μ_r :reversible permeability [H/m]	Fig. 1(b)
s : hysteresis parameter [Ω/m]	Fig. 1(c)
A : cross-sectional area [m^2]	48.6×10^{-6}
C : capacitance [F]	22.5×10^{-6}
l : flux path length [m]	78.3×10^{-3}
N : number of coil turns	180
R : resistance [Ω]	1.0



(a)



(b)

Fig. 3. Ferroresonant phenomenon. (a) Driving voltage v vs. time t. (b) Current i vs time t.

Finally, a state variable equation having a 3 $\times$ 3 matrix a is derived from Eq. (4).

$$\frac{d}{dt}\begin{pmatrix} \lambda \\ d\lambda/dt \\ q \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} \lambda \\ d\lambda/dt \\ q \end{pmatrix} + \begin{pmatrix} 0 \\ u_2 \\ u_3 \end{pmatrix} \quad \text{or} \quad \frac{d}{dt}\mathbf{x} = a\mathbf{x} + \mathbf{b}. \tag{5}$$

Where the elements $a_{21}, a_{22}, \cdots, u_2$ and u_3 are determined by Eq. (4). Solving Eq. (5) with a condition listed in Table 1 yields a typical ferroresonant phenonenon shown in Fig. 3. To solve the state variable equation, we employ the backward Euler method with automatic time-step modification. As shown in Fig. 3(a), the frequency of the driving voltage v is decreased from 70 to 33 Hz until time t =0.29s. Around this moment, the current i drastically increases as shown in Fig. 3(b). Therefore, the series ferroresonant circuit has been used for stabilized current supplies [5].

2.3. Chaotic behaviour in ferroresonance

Figure 4 illustrates i versus di/dt relationship, showing the chaotic behaviour while the frequency of driving voltage is fixed. To clarify the internal system condition, we calculate the characteristic values of the state transition matrix a in Eq. (5) in each of calcutation periods to consider the rule of this phenomenon. The characteristic value analysis could be applied to the linear system so that we assume this nonlinear system to be a piecewise linear system employing fine time-step widths for solving Eq. (5). Figures 5 and 6 illustrate the loci of characteristic values obtained from the state transition matrices, tracing on the same trajectories even though during chaos-like behaviour as in Fig. 4. Since the matrix a is a 3$\times$3 square matrix, then we have three characteristic values. Even though the chaotic phenomenon

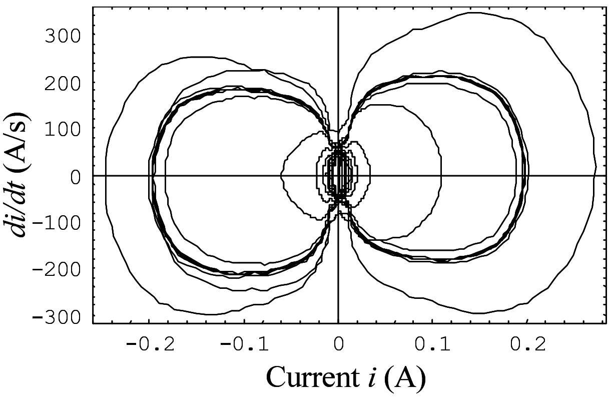


Fig. 4. Locus of i vs. di/dt.

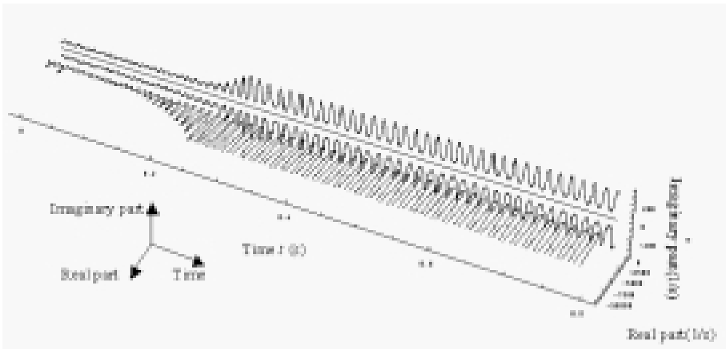


Fig. 5. Characteristic values time changing with time t.

is exhibiting, all of the characteristic values have a regular trajectory on the left half plane, meaning that the system keeps in stable state. Moreover, we extract the rule that the ferroresonant system has no hysteretic properties. Therefore, the chaotic mode is caused by the vector **b** in Eq. (5). In the vector **b**, there is an uncontrolable term having the nonlinear coefficient μ_r/s. This term causes the chaotic mode and is related to coercive force according to the term $(\mu_r/s)(dH/dt)$ in Eq. (1) [1]. Thus, it is clarified that the change of coercive force yields the ferroresonant phenomena accompanying with chaotic flicker. The Chua-type model makes it possible to represent and to extract the rule of chaotic phenomena in a quite efficient manner. This is because the parameters of the Chua model are independent of past magnetization histories, suggesting that such parameters are essentially required in the nonlinear calculations for prediction.

3. Conclusions

In this paper, we have studied the chaotic phenomena of series ferroresonant circuit to extract the rule in the chaotic system. We have derived the state variable equation of the circuit equations employing the Chua-type magnetization model. After that we have carried out the transient analysis of the ferroresonant circuit. The characteristic value analysis of the state transition matrices obtained in every calculation period of backward Euler method has clarified that the ferroresonant system works in stable state although

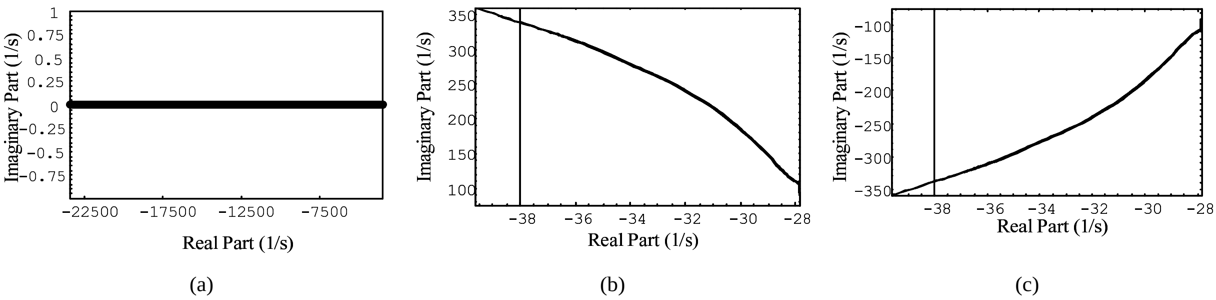


Fig. 6. Loci of the characteristic values changing. (a)-to-(c) correspond to large-to-small in characteristic values, respectively.

the chaotic flicking is accompanying. Moreover, we have been clarified that this chaotic flicking is closely related to the time changing of coercive force, i.e., magnetic relaxation aftereffect. This means that our approach with the Chua-type magnetization model enables us separating the nonlinear system into stable and instable terms in magnetization processes. This is because the parameters of the Chua-type magnetization model are measured independent of the past magnetization histories.

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