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Semi-analytical electromagnetic field computation

Yasuyuki Watazawa*, Seiji Hayano and Yoshifuru Saito
Graduate school of Engineering, Hosei University, 3-7-2 Kajino, Koganei, Tokyo 184-8584, Japan

Abstract. Any of the numerical methods for electromagnetic field analysis essentially require a subdivision of a problem region. By notifying this subdivision, we have previously proposed a new methodology by applying the analytical solution to each of the discretized parts. The first trial of this semi-analytical electromagnetic filed computational methodology has made it possible to simulate the complex electromagnetic field distributions not obtainable by the conventional numerical schemes, such as finite and boundary elements means. However, our semi-analytical method encounters some difficulty when analyzing the high frequency electromagnetic field distributions because of the displacement currents.In the present paper, we propose a new semi-analytical approach taking the displacement currents into account to overcome this difficulty.

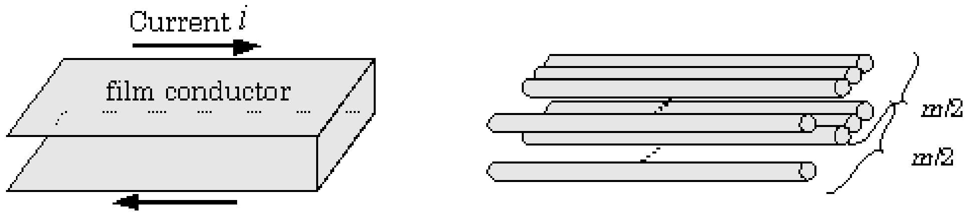
1. Introduction

A high-performance personal computer comes into wide use extensively due to those versatile functions and the low prices. So far, because of the high performance requirement, a high-speed computer carried out most of the numerical analysis of electromagnetic fields. However, widely spreading of high-performance personal computers in recent years has made it possible to carry out an electromagnetic field analysis in the individual level. Thereby, various commercial base software packages for electromagnetic field analysis are used in quit popular manner. To evaluate a numerical solution with higher accuracy, any of the numerical methods, e.g., finite and boundary elements, essentially force an enormous computation capability to the personal computers, because any numerical methods require a large number of discretization to the target problem region.

To overcome this difficulty, we have previously proposed a semi-analysis approach to the electromagnetic field analysis [1]. Let us consider a simple cylindrical coil, our method requires only one discretization because we employ an analytical solution. Thus, it is obvious that our semi-analytical approach reaches to analytical solution when employing a few discretization. However, our semi-analytical method encounters some difficulty when analyzing the high frequency electromagnetic field distribution because of the displacement currents. Thereby, we have to exploit a new modeling scheme of semi-analytical method in consideration of the displacement currents.

In this paper, we propose the new model that a displacement current was taken into consideration as a capacitance between the conductors.

*Corresponding author: Yasuyuki Watazawa, Graduate school of Engineering, Hosei University, 3-7-2 Kajino, Koganei, Tokyo 184-8584, Japan. Tel.: +81 42 387 6200; Fax: +81 42 387 6200; E-mail: watazawa@ysaitoh.k.hosei.ac.jp.



(a) Model (b) Subdivision of model

Fig. 1. Semi-analytical modeling of a film conductor.

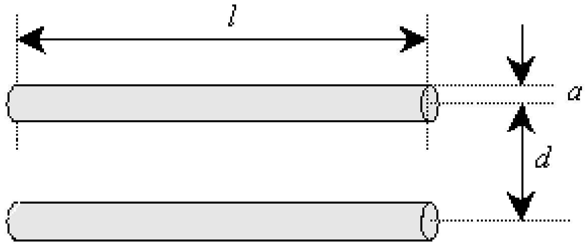


Fig. 2. Two of the m^{th} cylindrical conductors.

2. Semi-analytical method

As an example, let us consider the high frequency current distributions on a film conductor shown the Fig. 1(a). As shown in Fig. 1(b), the film conductor is subdivided into a large number of small cylindrical conductors.

The length of each subdivided cylindrical conductor is the same as those of the film conductor. When we assume the m^{th} cylindrical conductors, the sum of entire cross sectional-area of subdivided cylindrical conductors is equivalent to those of original film conductor. Figure 2 shows two of the m^{th} cylindrical conductors.

Since the shape of a subdivided small conductor in Fig. 2 is a simple cylindrical form, it is possible to apply the classical formulae for the inductance, resistance and capacitance calculations. Thus, the resistance r, self inductance L, mutual inductance $M_{ij}(i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, m, i \neq j)$ and capacitance $C_{ij}(i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, m, i \neq j)$ are respectively given by Eq. (1).

$$r = \sigma \frac{1}{\pi a^2}, \quad M_y = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{l + \sqrt{l^2 + d_{ij}^2}}{d_{ij}}\right) - \sqrt{1 + (\frac{d_{ij}}{l})^2} + \frac{d_{ij}}{l}\right], \tag{1}$$

$$L = \frac{\mu_0}{8\pi} l + \frac{\mu_0}{2\pi} l \left[\ln\left(\frac{2l}{a}\right) - 1\right], \quad C_{ij} = \frac{\varepsilon_0}{d_i j}\frac{S}{m}.$$

In Eq. (1), σ, a, l, μ_0 and S are the resistivity, permeability, length of a conductor, radius of the subdivided small conductor and cross-sectional area, respectively. The subscripts i and j refer to the i^{th} and j^{th} cylindrical conductors, respectively. Further, d_{ij} is a distance between the i^{th} and j^{th} cylindrical conductors. Thus, evaluation of the current distribution of the film conductor becomes a solution of an electric circuit shown in Fig. 3 in case of $m = 4$ [2].

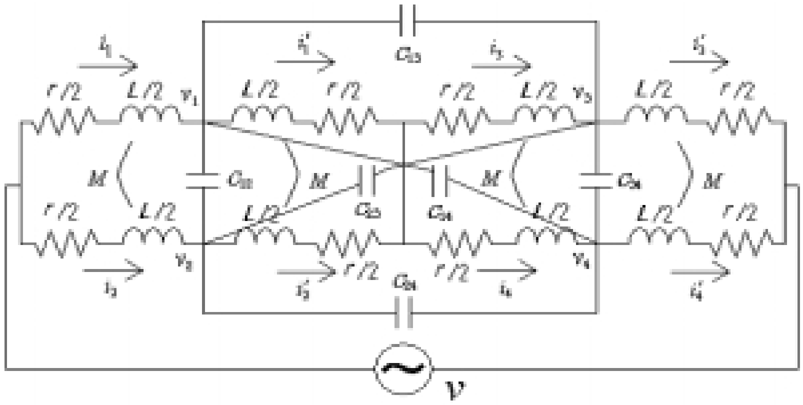
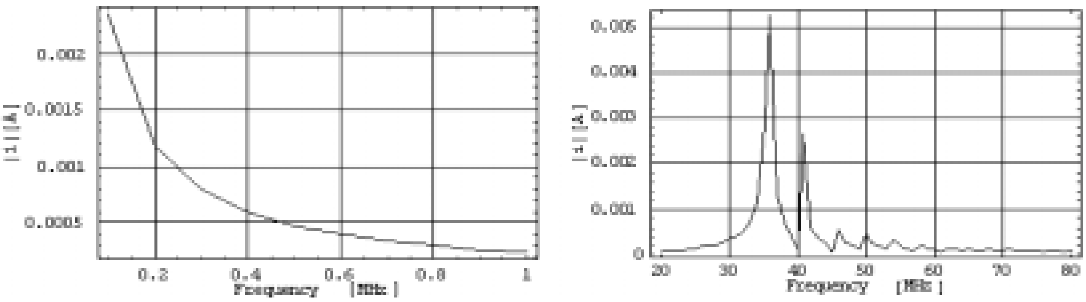


Fig. 3. Equivalent circuit of a film conductor where $m = 4$.



(a) 0.1~1.0[MHz] (b) 20~80[MHz]

Fig. 4. Entire current i vs. frequency characteristics.

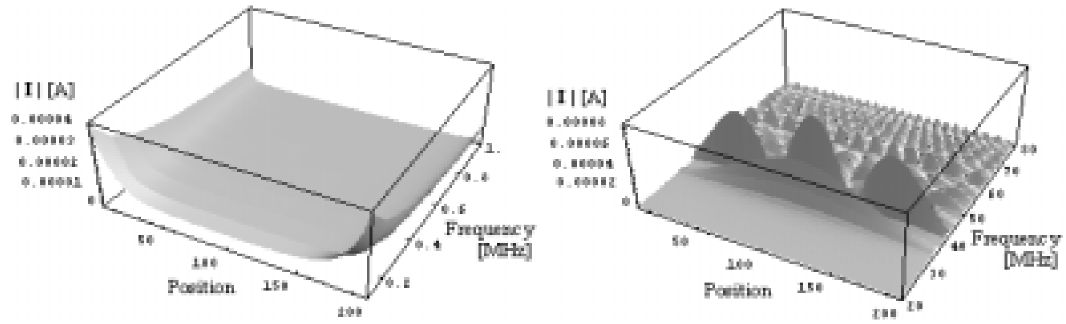
Table 1
Various constants used in the computation of a film conductor

Material	Copper
Resistivity	1.72×10^{-8}
Sizes (one side)	Width: 5 [cm] $\times$ Length : 30 [cm]
Distance between the films	0.5 [cm]
Radius of subdivided conductor	3.98×10^{-3} [cm]
Number of subdivisions (one side)	400 (200)
RMS value of the impressed voltage	1.0 [mV]

3. Examples

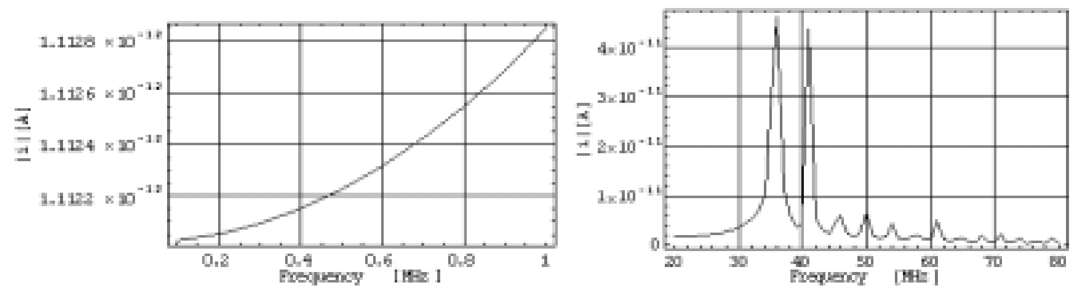
Let us compute the steady state current distribution in the film conductor shown in Fig. 1(a) when impressing the sinusoidal voltages. Table 1 lists various constants used in the computations. Figure 4 shows the entire current i vs. frequency characteristics. Figure 5 shows the current the distribution **I** vs. frequency characteristics.

Clearly, the resonance phenomena can be observed from the Fig. 4(b). Also, more than one resonance point exists. Generally, a magnetic field causes skin effect, and an electric field causes proximate effect.



(a) 0.1~1.0[MHz] (b) 20~80[MHz]

Fig. 5. Current distribution **I** vs. frequency characteristics.



(a) 0.1~1.0[MHz] (b) 20~80[MHz]

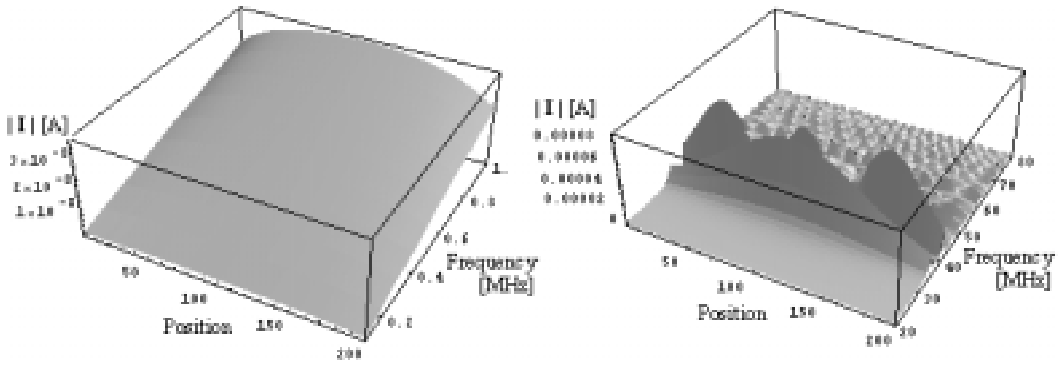
Fig. 6. Displacement current vs. frequency characteristics.

As shown in Fig. 5(a), skin effect forces to distribute the current on both edges of the film conductor at lower than 1 MHz frequency. On the other side, proximate effect forces to distribute the current on the central position of film conductor as shown in Fig. 5(b). Further, the current is distributed uniformly in the high-frequency region more than resonance frequency region. This means that a magnetic field dominants the system at low-frequency region and an electric field dominants the system at high-frequency region. Then, the resonance phenomenon occurs in the frequency band which both effects are interchanging.

Figure 6 shows the entire displacement current i vs. frequency characteristics. Figure 7 shows the displacement current distribution **I** vs. frequency characteristics. From the Figs 6(a) and Fig. 7(a), it is obvious that the displacement current grows up from low-frequency region when rising up the frequency. However, this displacement current i takes the maximum at the resonance frequency region. This means that major current flows through the capacitance between top and bottom films at the resonance frequency region.

4. Conclusion

As shown above, we have proposed a new semi-analytical approach taking the displacement currents into account for analyzing the electromagnetic field distribution. As a result, the skin as well as proximity



(a) 0.1~1.0[MHz] (b) 20~80[MHz]

Fig. 7. Displacement current distribution vs. frequency characteristics.

effects are computed by means of our semi-analytical method. Thus, we have succeeded in clarifying a physical mechanics of resonant phenomena by numerical simulation.

References

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