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# Magnetic fields control by single-sided exciting coils

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**Abstract.** In the present paper, we propose the methodologies in order to realize the desired magnetic field distributions by using single-sided exciting coils. As a result, it is revealed that a combination of the least squares and the generalized vector sampled pattern matching method suggests the practically realizable exciting coil layout.

## 1. Introduction

We try to control the magnetic field distribution at a flat square surface by means of the single side flat exciting coils to work out a magnetic domain-observing instrument under the controlled magnetic field distributions. Our designing methodology is reduced into solving for a linear system of equations: $\mathbf{Y} = C\mathbf{X}$, where $\mathbf{Y}$, $C$ and $\mathbf{X}$ are, respectively, the vector consisting of the desired magnetic field components, system matrix whose elements convert the exciting currents into the magnetic fields, i.e., spatial derivatives of Green function, and exciting current vector to be evaluated. In most case, the system matrix C becomes an ill-posed singular matrix. This means that we have to solve an ill-posed linear system of equations. This paper examines two numerical approaches in order to solve this ill-posed system of equations. As a result, it is revealed that a simple least squares method yields a reasonable solution even though the solutions are available in a limited physical constraint. Also, we have confirmed that the generalized vector sampled pattern matching (GVSPM, in short) method yields the reliable solutions under any hard physical constraints [1,2].

## 2. Design of a magnetic fields control device

### *2.1. System of equations*

Figure 1 shows a schematic diagram of our device for which movement of the magnetic domains is observed above the upper side surface. When the desired magnetic field vector $\mathbf{Y}$ is given, our problem is to compute the current vector $\mathbf{X}$ from this given magnetic field vector $\mathbf{Y}$. Denoting $\mathbf{C}$ as a system matrix whose elements are composed of the spatial derivatives of Green function, a system of equations is given by

$$\mathbf{Y} = \mathbf{C}\mathbf{X}. \tag{1}$$

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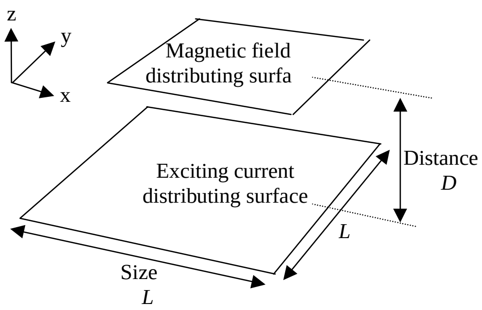



Fig. 1. Schematic diagram of the magnetic field distribution control device.

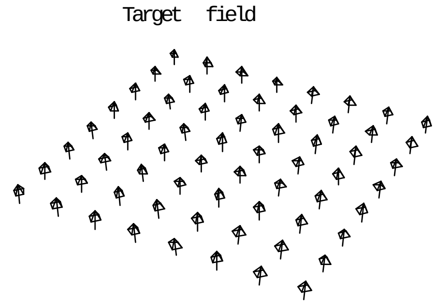



Fig. 2. Desired magnetic field distribution.

### 2.2. *Least squares solution*

In most case, a number of the elements in the vector $\mathbf{X}$ is not equivalent to those of the vector $\mathbf{Y}$ in Eq. (1). Namely, a number of unknowns $n$ is not equivalent to those of equations $m$, i.e., Eq. (1) is an ill-posed system of equations. When $n > m$, it is possible to apply the conventional least squares to Eq. (1), for which minimizes a least squares of error norm [3,4]:

$$\varepsilon = |\mathbf{Y} - C\mathbf{X}|, \tag{2}$$

a least squares solution vector $\mathbf{X}$ is given by

$$\mathbf{X} = (C^T C)^1 C^T \mathbf{Y}, \tag{3}$$

### 2.3. *Generalized vector sampled pattern-matching method*

#### 2.3.1. *Basic equations*

Least squares solution strategy is one of the deterministic methodologies to the ill-posed linear system of equations, but it is always required the condition $n > m$. To overcome this difficulty, we introduce the generalized vector sampled pattern matching method, which does not require any matrix inversion [1,2, 5].

Equation (1) can be rewritten as

$$\begin{aligned}
\mathbf{Y} &= \sum_{i=1}^{m} x_i \mathbf{C}_i, \\
C &= (\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3, \ldots \mathbf{C}_m), \\
\mathbf{X} &= (x_1, x_2, x_3, \ldots x_m)^T.
\end{aligned} \tag{4}$$

Equation (5) means that the normalized input vector $\mathbf{Y}'$ is always given by a linear combination of the weighted solutions $x_1 \frac{|\mathbf{C}_1|}{|\mathbf{Y}|}, x_2 \frac{|\mathbf{C}_2|}{|\mathbf{Y}|}, \ldots x_m \frac{|\mathbf{C}_m|}{|\mathbf{Y}|}$ with normalized column vectors .

$$\frac{\mathbf{C}_1}{|\mathbf{C}_1|}, \frac{\mathbf{C}_2}{|\mathbf{C}_2|}, \cdot \frac{\mathbf{C}_m}{|\mathbf{C}_m|} \cdots$$

That is

$$\frac{\mathbf{Y}}{|\mathbf{Y}|}=\sum_{i=1}^{m}\frac{|\mathbf{C}_i|}{|\mathbf{Y}|}x_i\frac{\mathbf{C}_i}{|\mathbf{C}_i|}=\sum_{i=1}^{m}x_i'\mathbf{C}'_i,$$
$$\mathbf{Y}'=\mathbf{C}'\mathbf{X}'. \tag{5}$$

Equation (4) means that the input vector $\mathbf{Y}$ is always given by means of a linear combination of the column vector $\mathbf{C}_i$. Therefore, defining an angle between the input vectors of $\mathbf{Y}$ and of $\mathbf{CX}^{(k)}$ given in terms of the $k^{\text{th}}$ iterative solution $\mathbf{X}^{(k)}$:

$$f[\mathbf{X}^{(k)}]=\frac{\mathbf{Y}}{|\mathbf{Y}|}\frac{C\mathbf{X}(k)}{|C\mathbf{X}^{(k)}|}=\frac{\mathbf{Y}}{|\mathbf{Y}|}\frac{|\mathbf{Y}|}{|\mathbf{Y}|}\frac{C'\mathbf{X}'^{(k)}}{|C'\mathbf{X}'^{(k)}|}=\mathbf{Y}'\frac{C'\mathbf{X}'^{(k)}}{|C'\mathbf{X}'^{(k)}|}, \tag{6}$$

when this objective function reaches to 1:

$$f[\mathbf{X}^{(k)}]\rightarrow 1, \tag{7}$$

it is possible to get the $k^{\text{th}}$ iterative solution $\mathbf{X}^{(k)}$ satisfying the normalized system of Eq. (5).

*2.3.2. Iterative solution scheme*

Consideration of Eq. (7) suggests a goal to look for that the Eq. (8) becomes a zero.

$$1-f[\mathbf{X}^{(k)}]=1-\mathbf{Y}'\frac{C'\mathbf{X}'^{(k)}}{|C'\mathbf{X}'^{(k)}|}\cdot \tag{8}$$

Multiplying the normalized input vector $\mathbf{Y}'$ to both sides of Eq. (8) leads to

$$\mathbf{Y}'-\frac{C'\mathbf{X}'^{(k)}}{|C'\mathbf{X}'^{(k)}|}=0. \tag{9}$$

Denoting $\mathbf{X}'^{(0)}$ as an initial vector of $\mathbf{X}'$, we have a following relationship:

$$\Delta\mathbf{Y}'^{(1)}=\mathbf{Y}'-\frac{C'\mathbf{X}'^{(0)}}{|C'\mathbf{X}'^{(0)}|}=C'\Delta\mathbf{X}'^{(1)} \tag{10}$$

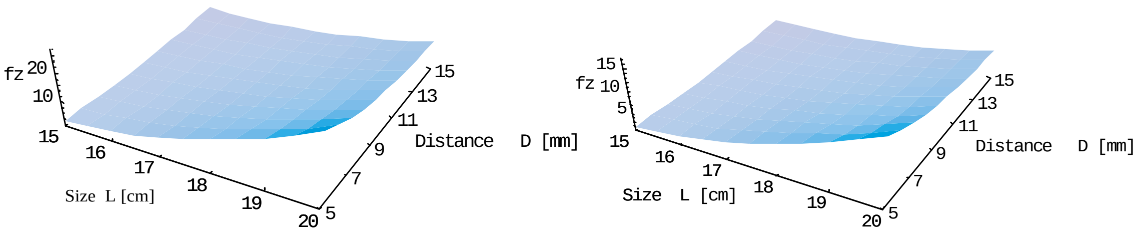
Generalization of Eq. (10) to the $k^{\text{th}}$ iterative solution $\mathbf{X}'^{(k)}$ of the GVSPM method yields

$$\mathbf{X}'^{(k)}=\mathbf{X}'^{(k-1)}+C'^{T}\Delta\mathbf{Y}'^{(k-1)}=C'^{T}\mathbf{Y}'+\left[I_m-\frac{C'^{T}C'}{|C'\mathbf{X}'^{(k-1)}|}\right]\mathbf{X}'^{(k-1)}, \tag{11}$$

where $I_m$ denotes a unit square matrix with order $m$.

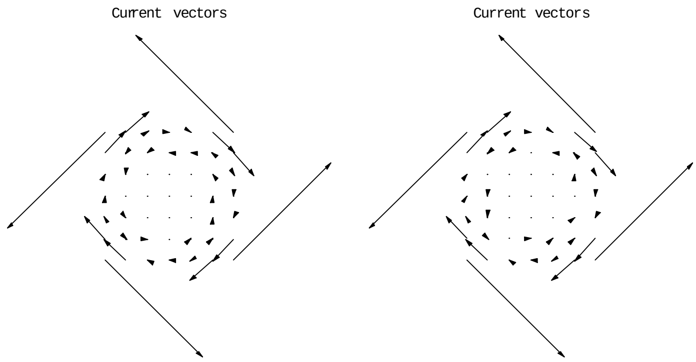
## 3. Example

Let us compute a current distribution flowing on the lower side surface in Fig. 1 when the desired magnetic field distribution shown in Fig. 2 at the upper side square surface is given. To carry out this



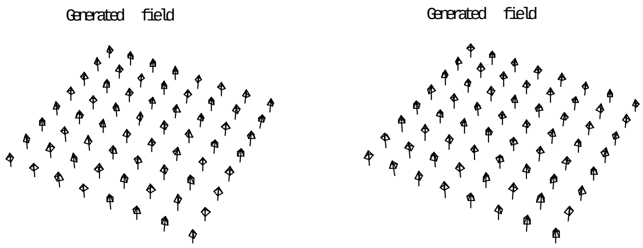

(a) Least squares (b) GVSPM

Fig. 3. Objective functions $fz$ in Eq. (12).




(a) Least squares (b) GVSPM

Fig. 4. Computed current distributions.




(a) Least squares (b) GVSPM

Fig. 5. Computed magnetic field distributions.

computation, it is essential to define an objective function, which reveals fitness between the desired and computed magnetic field distributions. In this paper, we employ a following objective function:

$$f_z = \frac{|\mathbf{H}_1|}{|\mathbf{H}_x| + |\mathbf{H}_y| + \sum_{i=2}^{k} |\mathbf{H}_i|}, \tag{12}$$

where $|\mathbf{H}_x|$, $|\mathbf{H}_y|$ are the norms of x- and y-magnetic field components; $|\mathbf{H}_1|$, $\sum_{i=2}^{k} |\mathbf{H}_i|$ are the desired and undesired magnetic field components in the direction of z-axis in Fig. 1, respectively. Classification of the desired and undesired magnetic field components can be carried out by means of the discrete wavelets

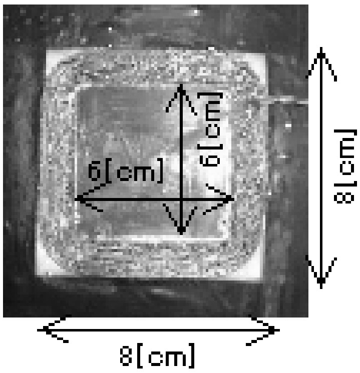



Fig. 6. Tested exciting coil.

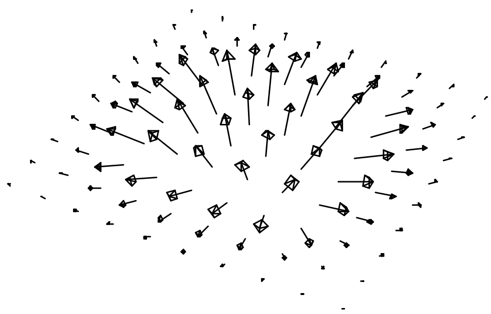
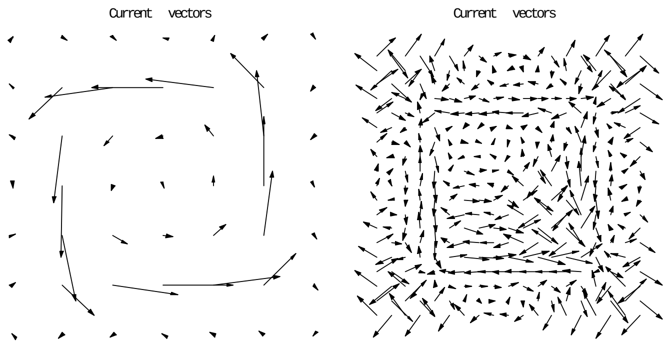


Fig. 7. Magnetic field distribution caused by a tested coil in Fig. 6.



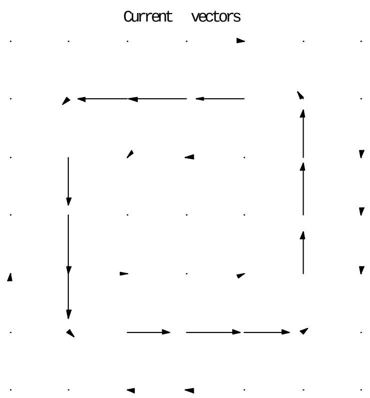

(a) Least squares (b) GVSPM

Fig. 8. Computed current distributions.

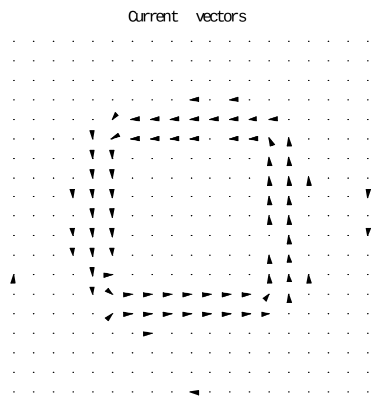
transform using Daubechies 2nd order base function [6,7]. We computed the current distributions by the least squares as well as GVSPM methods when changing the distance D from 5 to 15 mm and the size L from 15 to 20 cm in Fig. 1. Figure 3 shows the objective functions. In this figure, the left and right figures are the least squares and GVSPM objective functions, respectively. Similarity between the left and right figures in Fig. 3 reveals that both of the least squares and GVSPM give the similar values of D and L. The maximum value 27.62 of the objective function Eq. (12) was obtained at the condition of $D = 5$ mm and $L = 20$ cm by the least squares. On the other side, the maximum value 16.67 of the objective function was obtained at the condition of $D = 5$ mm and $L = 20$ cm by the GVSPM method. Even though the values of objective function are different each other, both the least squares and GVSPM methods give the same design parameters $D = 5$ mm and $L = 20$ cm. Figure 4 and 5 show the computed current and magnetic field distributions, respectively. By observing the results in Figs 4 and 5, it is obvious that both of the least squares and GVSPM methods give the similar distributions.

### 3.1. Experimental verification

To verify our approach experimentally, we have worked out a sample coil shown in Fig. 6. Figure 7 shows the magnetic field distribution caused by this coil when impressing alternating currents. If this magnetic field distribution is given as a desired magnetic field distribution, then the least squares and GVSPM methods yield the current distributions shown in Fig. 8. Any of the current distributions in Fig. 8




(a) Least squares




(b) GVSP

Fig. 9. Current distributions after convolution.

contain a lot of noisy current vectors. To remove these noisy current vectors and to extract the major dominant current vectors, we convolved the normalized current vectors which had been independently computed from the x-, y-, and z- magnetic field components in Fig. 7. It must be noted that a convolution between the normalized currents taking their maximum 1 extracts the common currents [5,8]. Figure 9 shows the extracted common major current distributions by the convolving the normalized current vectors. Observation of the current distribution in Fig. 9 reveals that the least squares method gives the outline distribution of exciting coil in Fig. 6 and GVSPM method gives the surface distribution. Thus, we have succeeded in verifying our methodologies.

## 4. Conclusion

We have proposed the magnetic field control methodology based on the inverse approach. As an initial trial, we have tried to realize a desired magnetic field distribution by the single side planner current carrying coils. As a result, it has been confirmed that our approach gives the practically realizable result. By means of the simulation as well as experiment, the validity of our method has been verified.

## References


[1] K. Yoda et al., *Dose optimization of proton and heavy ion therapy using generalized sampled pattern matching*, Phys. Med. Biol, IOP Publishing, 1997.

[2] I. Marinova et al., *Estimation of Current Distribution within Conductors by Field Measurements*, INVERSE PROBLEMS IN ENGINEERING MECHANICS, Mar, 1998.

[3] H. Takahashi, S. Hayano and Y. Saito, *Visualization Of The Currents On The Printed Circuit Boards, IEEE Visualization 1999, Late Breaking Hot Topics*, Oct, 1999, pp. 37–40.

[4] K. Wakabayashi, S. Hayano, Y. Saito, T.L. Kunii, K. Horii and M. Sakuma, *The Eigen pattern of graphics and its application to magnetic fields*, MAG-99-14, Nov, 1999, pp 15–18.

[5] D. Sekijima, S. Hayano, Y. Saito, A. Sawada and K. Horii, *Estimation of 2D Current Distributions by the Generalized SPM Method*, MAG-99-151, Nov, 1999, pp. 25–29.

[6] Y. Saito, *wavelet transform by Mathematica*, in Japanese, Asakura Publishing co., Sep, 1996.

[7] Y. Saito, *Fundamentals and applications of wavelets transform*, in Japanese, Asakura Publishing co., Apr, 1998.

[8] S. Miyahara, S. Hayano and Y. Saito, *A Measurement of the Quasi-3Dimensional Current Distribution and its Application*, MAG-99-15, Nov, 1999, pp. 35–39.