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FLOW IMAGE COMPRESSION USING WAVELETS

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ABSTRACT

The wavelet compression technique was applied to turbulent image processing for reducing physical storage and extracting the compact dominant features in this study. The two compression methods, called zone and threshold compression, were employed. It was found that a high order wavelet basis provided good compression performance for compressing turbulent images and two compression methods exhibited almost same performance. It was realized that the compressed image had both lower compress ratio and larger correlation coefficients. By changing compression ratio the compressed images exhibited different scale structures in turbulent jet. This indicated clear that large-scale structure dominates the jet.

INTRODUCTION

The turbulent jets exhibit complex structure with a wide range of coexisting scales and a variety of shapes in the dynamics. To understand the turbulent mixing process, various measures of the isosurface geometry from image appeared from 1980's. Catrakis and Dimotakis (1996) reported two-dimensional, spatial measurements of the jet-fluid concentration field in liquid-phase. Recently, Li et al. (1998, 1999) developed an application of two-dimensional orthogonal wavelets to turbulent image analysis. The multiresolution turbulent structures and the coherent structure were extracted and visualized. It is well known fact that some correlation between neighboring elements exists in turbulent image, and indicates that image contains the redundancy of information in some extent. This redundancy allows us to compress the turbulent image for economy in storing, transmitting or further processing with high speed and to extract the compact dominant features.

The International Standard Organization (ISO) has proposed the JPEG standard (Wallace, 1991) for still image compression and MPEG standards (Gall, 1991) for video compression. These standards employ discrete cosine transform (DCT) to reduce the spatial redundancy present in the images or video frames. We note that DCT has the drawbacks of blocking artifacts, mosquito noise and aliasing distortions at high compression ratios (Wallace, 1991; Coifman et al., 1992). However, this method that was often used in turbulent image is only to eliminate the low intensity pixels of image file. Because the low intensity pixels contribute little information about turbulent structure, this type of image compression has very little significant.

Over the past decade discrete wavelet transform (DWT) has emerged as a popular technique for image processing (Mallat, 1989). DWT has high decorrelation and energy compaction efficiency. The blocking artifacts and mosquito noise are absent in a wavelet-based code due to their overlapping basis functions. The aliasing distortion can be reduced with a proper choice of wavelet filters. In addition, the basis functions are localized in both the spatial and frequency domains. Hence, they are better matched to the human visual system (HVS) characteristics. Generally, a wavelet providing optimal performance for the whole image is selected.

Although a wide variety of wavelet-based image processing has been reported in the literature, few applications can be found in the area of fluid mechanics. Recently, Li et al. (1999) improved spatial resolution and reliability in the particle image velocimetry (PIV) system using the wavelet image compression technique.

In this paper there are two motives to develop an application of wavelet compressing technique to turbulent image. One is to minimize the physical storage, and the other is to develop a visualized tool for extracting compact dominant features from turbulent images. Certainly, the noise in images can also be reduced.

In wavelet-based image compression, the compression performance depends on the choice of wavelets. To investigate the performance of the different compression methods and choice of the wavelet bases, we first apply two wavelet compression techniques with Coifman and baylkin wavelet bases to the digital-imaging photograph of turbulent jet. Then, the dominant features of turbulent structures are discussed when changing compression ratio of compressing images.

WAVELET COMPRESSION TECHNIQUE

Let us consider a two-dimensional scalar field $f(\bar{x})$ and isotropic mother wavelet $\psi(\bar{x})$ by treating $\bar{x}=(x_1,x_2)$ as vector. The family of wavelets $\psi_{\bar{b},a}(\bar{x})$, which is translated by position parameter $\bar{b}\in R^2$ ($\bar{b}=(b_1,b_2)$) and dilated by scale parameter $a\in R^+$, is written as

$$\psi_{\bar{b},a}(\bar{x})=\frac{1}{a}\psi\left(\frac{\bar{x}-\bar{b}}{a}\right). \qquad (1)$$

The two-dimensional continuous wavelet transform of $f(\bar{x})$ can be defined as

$$\begin{aligned} Wf(\bar{b},a) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\bar{x})\psi_{\bar{b},a}(\bar{x})d^2\bar{x} \\ &= \frac{1}{a}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\bar{x})\psi\left(\frac{\bar{x}-\bar{b}}{a}\right)d^2\bar{x} \end{aligned} \qquad (2)$$

The coefficients of continuous wavelet transform $Wf(\bar{b},a)$ can be interpreted as the relative contribution of scale a to the scalar field $f(\bar{x})$ at position $\bar{b}$.

If the wavelet is admissible, the inversion formula is

$$\begin{aligned} f(\bar{x}) &= \frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} a^{-3}Wf(\bar{b},a)\psi_{\bar{b},a}(\bar{x})da d^2\bar{b} \\ &= \frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} a^{-4}Wf(\bar{b},a)\psi\left(\frac{\bar{x}-\bar{b}}{a}\right)da d^2\bar{b} \end{aligned} \qquad (3)$$

It is well known that black-and-white images are often used to describe turbulent structures, and are expressed in a discrete numerical form as a function $f(x_1,x_2)$ over two dimensions in which the function value $f(x_1^0,x_2^0)$ represents the "gray scale" value of the image at the position or pixel values (x_0,y_0). Therefore, we must considered to use the discrete type of wavelet transform. In the discrete wavelet transform, the dilation parameter a and the translation parameter $\bar{b}$ both take only discrete values in Eq. (2). For any scale a we choose the integer (positive and negative) powers of one fixed dilation parameter $a_0>1$, i.e., a_0^m, and different values of m correspond to wavelets of different widths. It follows that the discretization of the translation parameter $\bar{b}$ should depend on m: narrow wavelets (high frequency) are translated by small steps in order to cover the whole field, while wider wavelets (lower frequency) are translated by large steps. Since the width of is proportion to a_0^m, we choose therefore to discretize $\bar{b}$ by $\bar{b}=\bar{n}b_0a_0^m$, where b_0 is fixed. Starting from wavelet basis $\psi_{m,n}(x)=a_0^{-m/2}\psi(a_0^{-m}x-nb_0)$ the corresponding discretely family of wavelets is simply to take the tensor product functions generated by two one-dimensional bases:

$$\Psi_{m_1,n_1;m_2,n_2}(x_1,x_2)=\psi_{m_1,n_1}(x_1)\psi_{m_2,n_2}(x_2). \qquad (4)$$

For some very special choices of $\psi(x)$ and a_0, b_0, the $\psi_{m,n}(x)$ constitutes an orthonormal basis. In particular, if we choose a_0=2, b_0=1, then there exist $\psi(x)$, with good physics-frequency localization properties, such that

$$\Psi_{m_1,n_1;m_2,n_2}(x_1,x_2)=2^{-(m_1+m_2)/2}\psi(2^{-m_1}x_1-n_1)\psi(2^{-m_2}x_2-n_2). \qquad (5)$$

constitutes an orthogonal basis. In this basis the two variables x_1 and x_2 are dilated separately. The oldest example of a function $\psi(x)$ for which the $\psi_{m,n}(x)$ constitutes an orthogonal basis is the Haar function, constructed long before the term "wavelet" was coined. In the last ten years, various orthogonal wavelet bases, e.g., Meyer basis, Daubechies basis, Coifman basis, Battle-Lemarie basis, Baylkin basis, spline basis, and others, have been constructed. They provide excellent localization properties in both physical and frequency spaces.

Therefore, the two-dimensional discrete wavelet transform is given by

$$Wf_{m_1,n_1;m_2,n_2}=\sum_i\sum_j f(x_1^i,x_2^j)\Psi_{m_1,n_1;m_2,n_2}(x_1^i,x_2^j). \qquad (6)$$

The reconstruction of the original scalar field can be achieved by using

$$f(x_1,x_2)=\sum_{m_1}\sum_{m_2}\sum_{n_1}\sum_{n_2}Wf_{m_1,n_1;m_2,n_2}\Psi_{m_1,n_1;m_2,n_2}(x_1,x_2). \quad (7)$$

The total energy of the scalar field is given by summing over all scales and components as following

$$\sum_{i,j}\left(f\left(x_1^i,x_2^j\right)\right)^2=\sum_{m_1}\sum_{m_2}\sum_{n_1}\sum_{n_2}\left(Wf_{m_1,n_1;m_2,n_2}\right)^2. \quad (8)$$

In general, the energy of image field is highly concentrated in a small number of wavelet coefficients. The image compression is usually defined as the representation of image using fewer basis function coefficients than were originally given, either with or without loss of information. The basic method to compress the image based on wavelets is to setting wavelet coefficients of modes with insignificant energy to zero. There are usually following two methods. One way, called the *zone compression method* (ZCM), can be summarized in three steps:

(1) Compute wavelet coefficients $Wf_{m_1,n_1;m_2,n_2}$ representing an image in orthonormal wavelets basis.
(2) Specify the number of wavelet coefficients M to retain, that is, fix the compression ratio M/N where N is the total number of wavelet coefficients before compression and delete all other wavelet coefficients.
(3) The image is reconstructed from nonzero wavelet coefficients based on inverse wavelet transform.

Another way is called the *threshold compression method* (TCM), and the procedure has following steps:

(1) Compute wavelet coefficients $Wf_{m_1,n_1;m_2,n_2}$ representing an image in orthonormal wavelets basis.
(2) Fix the threshold C, so that any wavelet coefficient is set to zero if $\left|Wf_{m_1,n_1;m_2,n_2}\right|<C$. If the number of nonzero wavelet coefficients is M, the compression ratio may be determined by M/N.
(3) Reconstruct the image based on nonzero wavelet coefficients using inverse wavelet transform.

We can then adjust the number of wavelet coefficients M or the threshold C to vary the compression ratio and to extract dominant structure. For evaluating the compressed feature, the correlation coefficients between the original image and compressed image is employed in this paper.

CHOICE OF WAVELET BASES

The choice of wavelet basis and its order is important in achieving good compressed performance. In order to investigate the effect of different orthogonal wavelet basis on the compressing flow image, the following sets of compactly supported orthonormal wavelets are used in this study.

(1) Coiflets wavelet with orders 18, 24 and 30;
(2) Baylkin wavelet with orders 6, 12 and 18.

These orthogonal wavelet bases have respectively their families that are defined by different index number or the number of wavelet's coefficients. As the index number or order, i.e., the number of wavelet's coefficients, increases, the wavelet becomes smoother closer to a smoothly windowed harmonic function, and the wavelet's Fourier transform becomes increasingly compact, i.e., are compact in the frequency domain.

In order to evaluate the compression feature and study the dominant features of turbulent image, the digital-imaging photograph of turbulent jet that were experimentally obtained by Catrakis and Dimotakis (1996) is used in this paper. As described in Catrakis and Dimotakis' paper (1996), experiments were carried out in liquid-phase turbulent-jet flows, and images of slices, which relied on laser-induced fluorescence digital-imaging techniques, through the three-dimensional scalar filed of round momentum-driven turbulent jets were obtained. Transverse sections in the far field of the jet, at downstream position z/d=275 (jet-nozzle diameter d is 2.54mm), were recorded on a cryogenically cooled 1024x1024 pixels CCD camera.

The black-and-white image of jet slice with Re=4.5x10^3 in Fig.1 is expressed in a numerical form as a function $f(x_1,x_2)$ over two dimensions in which the function value $f\left(x_1^0,x_2^0\right)$ represents the "gray scale" value of the image at the position or pixel values (x_0,y_0). The "gray scale" values are then normalized to one.

Fig.1 Image of a turbulent jet, 256X256 pixels with 8-bit grayscale

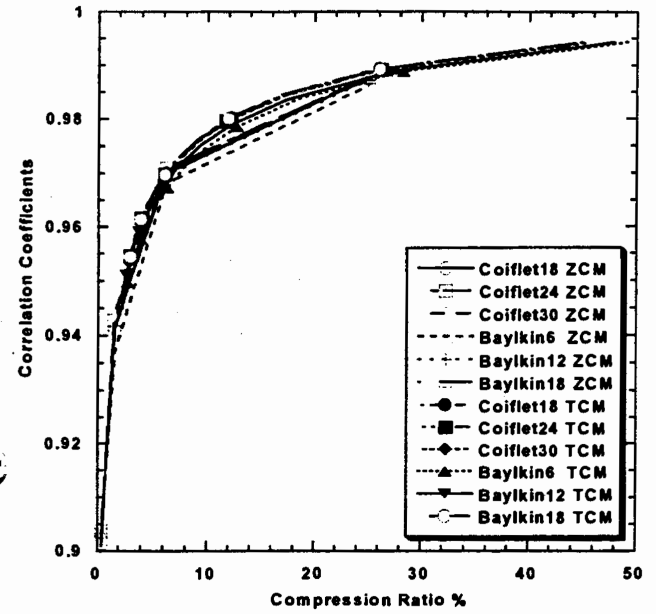

Fig.2 Compression performance for two compression methods with various wavelet bases

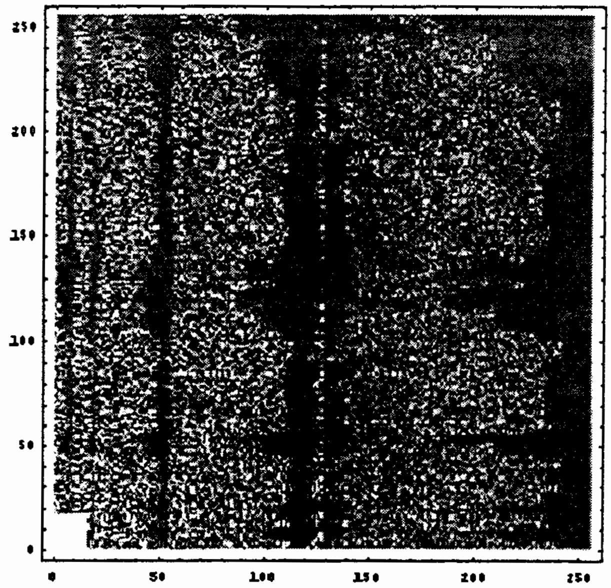

Fig.3 Wavelet coefficients of jet image based on wavelet bases Coiflet30

Figure 2 summarizes the performance of various wavelet families' bases and orders when compressing jet image of Fig.1 based on the zone compression and threshold compression methods (abscissa: compression ratio, ordinate: correlation coefficients). These two methods exhibit almost the same compression performance. It is evident that as increasing compression ratio, the correlation coefficients decreased and dropped quickly near compression ratios 6.25% and 1.56%. These two points may indicate important compression ratios in the turbulent image. It may be also found in Fig.2 that the best subjective performance was obtained with high order wavelet bases (Coiflets wavelet with orders 18 to 30 and Baylkin wavelet with order 18), because a high order wavelet base can be designed to have good frequency localization that in turn increases the energy compaction. The regularity of wavelet also increases with its order. In addition, more vanishing moments can be obtained with a higher order wavelet base. On the other hand, although a lower order wavelet base is expected to have a better time localization and therefore preserve the crucial edge information, lower order wavelet bases for compressing turbulent images show smaller correlation coefficients than that of higher order wavelet bases at same compression ratios.

In the following, we only discuss the features of the compressed image using the Coiflets basis of order 30. To study how the energy of the turbulent image is distributed, wavelet coefficients $Wf_{m_1,n_1;m_2,n_2}$ of image (Fig.1) are plotted in Fig.3 (abscissa and ordinate: levels, the white and black: wavelet coefficients). The highest wavelet coefficients is displayed as a white and the lowest as black. It is observed that the larger

Fig. 4 Compressed jet image with compression ratio 45.6% and correlation coefficients 0.994 based on the threshold compression method with Coiflets wavelet of order 30

magnitude of $Wf_{m_1,n_1;m_2,n_2}$ or energy of the image concentrates in roughly the smaller level range. So we can use wavelets to do better compression.

VISUALIZATION OF TURBULENT STRUCTURE

Figure 4 shows a compressed image based on the threshold compression method with compress ratio 45.6% and correlation coefficients 0.994, which is realized by fixing the threshold of gray scale value C=0.1. At this compression ratio the redundancy of information and insignificant information on the small-scale structure are reduced. The image may almost keep the same spatial resolution as original image.

Figures 5-7 shows a sequence of reconstructed compression images based on the zone compression method, which differ in the number of wavelet coefficients that have been kept. These images are reconstructed from the remaining 25%, 6.25% and 1.56% of the 65536 wavelet coefficients and exhibit higher correlation coefficients with 0.988, 0.971 and 0.944, respectively. These correlation coefficients of compressed turbulent images show higher compression performance than that of PIV images (Li et al., 1999). From the spectra analysis of the compressed images, it is found that three compressed images represent turbulent structures within the range of scale 0.8~14.3*mm*, 3.6~14.3*mm* and 7.2~14.3*mm*, respectively. Comparing Fig.5 with Fig.4, the higher correlation coefficient is kept, although the compression ratio becomes lower. This image keeps still a relatively high spatial resolution that closes to the original image, and shows the turbulent structure with scale 0.8~14.3*mm*. For reducing compression ratio further, Figure 6 exhibits the medium-scale structure with scale 3.6~14.3*mm*. Peaks appearing in the image correspond to positions of eddies. This image may be described as the "zoom-out" of Fig.5. When compression ratio is decreased to 1.56%, large-scale structure of 7.2~14.3*mm* can be observed in Fig.7. Corresponding to the physically intuitive of physics of the flow, peaks in Fig.7 imply large-scale eddies. They are the uppermost and energy-containing vortices. From the larger correlation coefficient in Fig.7, it can say that large-scale structure dominates the jet and significant flow information is still kept. From above, these compressed images provide further evident of multi-scale structures in turbulent jet, and significant information on the turbulent structure is kept in the compressed image.

At last we discuss the compressed image with help of lower order wavelet basis. Figure 8 shows a compressed image based on Baylkin wavelet basis of order 6. This image is reconstructed from the remaining 1.56% of the 65536 wavelet coefficients and has correlation coefficients of 0.937. It is evident that the image shows non-smoothness distribution and information on the large-scale structure is hardly obtained. This is because Baylkin wavelet basis of order 6 is nowhere differentiable, although it is continuous and compact in the physical domain. Therefore, it is very important that orthonormal wavelet basis must be smoothness function when compressing or analyzing

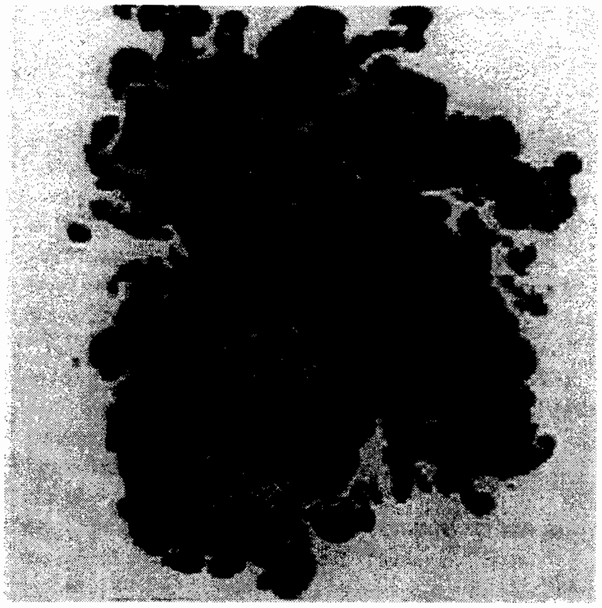

Fig.5 Compressed jet image with compression ratio 25% and correlation coefficients 0.988 based on the zone compression method with Coiflets wavelet of order 30

Fig.6 Compressed jet image with compression ratio 6.25% and correlation coefficients 0.971 based on the zone compression method with Coiflets wavelet of order 30

the turbulent image.

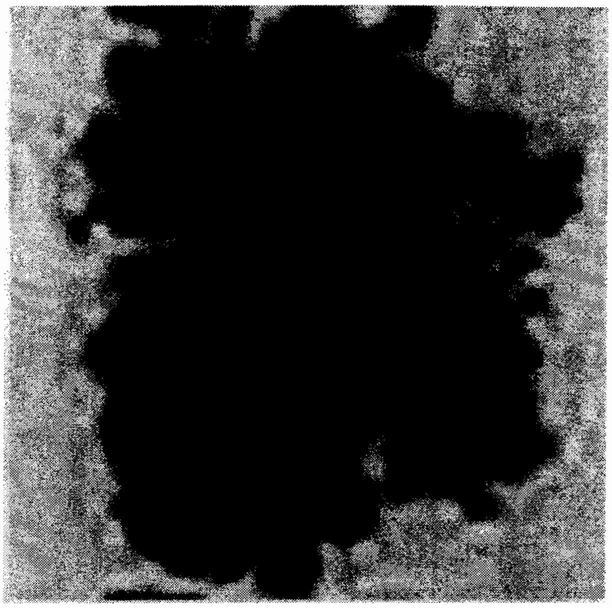

Fig.7 Compressed jet image with compression ratio 1.56% and correlation coefficients 0.944 based on the zone compression method with Coiflets wavelet of order 30

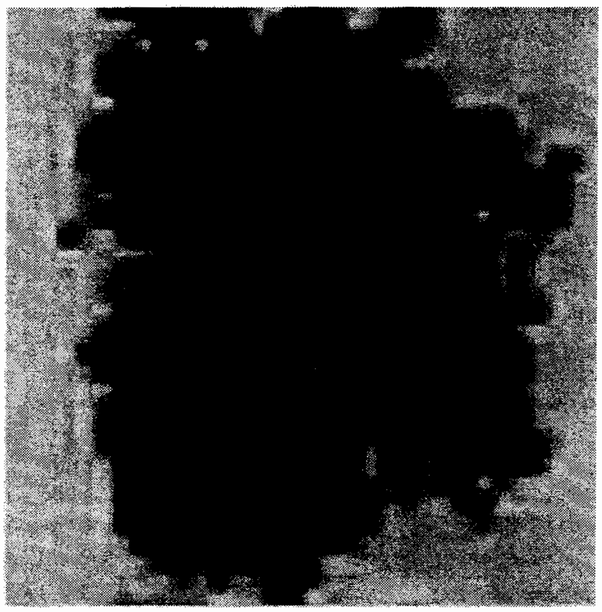

Fig.8 Compressed jet image with compression ratio 1.56% and correlation coefficients 0.937 based on the zone compression method with Baylkin wavelet of order 6

CONCLUSIONS

The wavelet compression technique was applied to turbulent jet processing for reducing physical storage and extracting the compact dominant features in this study. The following main results are summarized.

(1) A high order wavelet basis provides good compression performance for compressing turbulent images, because they have good frequency localization that in turn increases the energy compaction.

(2) The zone and threshold compression methods exhibit almost same performance, and the compressed image with lower compress ratio and larger correlation coefficients is obtained.

(3) Significant information on the turbulent structure is kept in the compressed image.

(4) By changing compress ratio the compressed images exhibit turbulent structures in different broader scales.

REFERENCES

H. J. Catrakis and P. E. Dimotakis, 1996, "Mixing in Turbulent Jets: Scalar Measures and Isosurface Geometry", *J. Fluid Mech.*, Vol.317, pp.369-406.

R. Coifman, Y. Meyer, S. Quake and V. Wickerhauser, 1992, "Entropy based algorithms for best basis selections", *IEEE Trans. on Information Theory*, 38(2), pp.713-718.

D. L. Gall, 1991, "MPEG: A video compression standard for multimedia applications", *Communications of the ACM*, 34(4), pp.46-58.

H. Li, M. Takei, M. Ochi, Y. Saito and K. Horii, 1998, "Effect of Different Orthogonal Wavelet Basis on Multiresolution Image Analysis of a Turbulent Flow", *Proceedings of The International Conference on Optical Technology and Image Processing in Fluid, Thermal and Combustion Flow*, No.AB015, pp.1-11.

H. Li, M. Takei, M. Ochi, Y. Saito and K. Horii, 1999, "Application of Two-dimensional Orthogonal Wavelets to Multiresolution Image Analysis of the Turbulent Jet", *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol.42, No.137.

H. Li, M. Takei, M. Ochi, Y. Saito and K. Horii, 1999, "Choice of Wavelets for PIV Image Compression", *FEDSM99-7063*, pp.1-6.

S. G. Mallat, 1989, "A theory for multiresolution signal decomposition: the wavelet representation", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 11(7), pp.674-693.

G. K. Wallace, 1991, "The JPEG still picture compression standard", *Communications of the ACM*, 34(4), pp.30-45.