INVERSE PROBLEMS IN ENGINEERING MECHANICS
M. Tanaka, G.S. Dulikravich (Eds.)



# ROTATIONAL AND DIVERGENT COMPONENTS IDENTIFICATION OF VECTOR FIELDS BY THE MINIMUM NORM METHOD

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## ABSTRACT

In the present paper, a method for classifying a rotational and a divergence components out of an observed vector is proposed. Our new method is applied to analysis of electric current flows on a two-dimensional field. Further, a post processing for an estimated current distribution is carried out by our method. This makes it possible to searching for the eddy currents as well as the leakage magnetic field sources on a printed circuit board (PCB). As a result, numerical and experimental examples demonstrated the validity of our method for classifying source current components out of an estimated current distribution.

## KEYWORDS

Helmholtz theorem, minimum norm method, eddy current, leakage magnetic field source

## INTRODUCTION

Recently, magnetic field source searching problems lays as one of extremely important engineering problems, such as the nondestructive testing or the electromagnetic compatibility (EMC) problems. But an observed vector field, such as electric current or magnetic fields, mostly contains more or less noise vectors [1]. In order to reduce the effect of noise, one of the ways is to carry out the post processing for the estimated magnetic field sources, i.e., current vector distributions. After classifying the current vectors into rotational and divergent components, unnatural components in the system can be removed by this post processing.
This paper proposes a post processing method which classifies the source component vectors to be identified. Key idea is to classify the rotational and divergent vector fields. By means of the Helmholtz theorem, any vector can be represented as the summation of rotational and divergent

vector fields [2,3]. To evaluate the rotational and divergent fields, it is essential to solve an inverse problem whose solution is composed of the scalar and vector potentials. The solution methodology to this inverse problem is the minimum norm method. A rotational vector field can be obtained by taking a rotation of vector potentials. Also, a divergent vector field can be obtained by taking a gradient of scalar potentials.

At first section, an inverse approach for identifying both of unknown scalar and vector potentials out of known two-dimensional current fields is proposed. At second section, the numerical simulations are carried out to check up the validity of our method. Finally, our method is applied to the practical problems, which are concerning with the searching for eddy current and source current from the locally measured magnetic fields.

## ROTATIONAL AND DIVERGENT VECTOR FIELDS IDENTIFICATIONS

### *Frame Equation*

According to the Helmholtz Theorem [3], an arbitrary vector $\mathbf{F}$ can be expressed by

$$\mathbf{F} = \nabla \times \mathbf{V} + \nabla \phi, \tag{1}$$

where $\mathbf{V}$ and $\phi$ are vector and scalar potentials, respectively. Eq.(1) means that an arbitrary vector field is composed of the rotational and divergent vector fields. A rotational vector field can be obtained by taking a rotation of vector potentials $\mathbf{V}$. Also, a divergent vector field can be obtained by taking a gradient of scalar potentials $\phi$.

### *Two-Dimensional Vector Fields*

Let us consider a potential (e.g. vector $V_z$ and scalar $\phi$ in Fig.1) evaluation problem when the vectors (e.g. $\mathbf{F}$ in Fig.1) are given. Hence, we consider here an inverse problem for identifying both of unknown scalar and vector potentials out of known vectors in two-dimensional fields. In two dimensional vector fields, the rotational and divergent components are given by

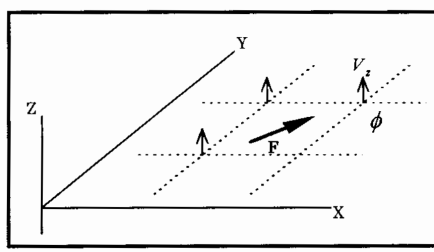

Fig.1. Two-dimensional model.

$$\nabla \times \mathbf{V} = \begin{bmatrix} \mathbf{i}_x & \mathbf{i}_y & \mathbf{i}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0 & V_z \end{bmatrix}, \tag{2}$$

$$= \mathbf{i}_x \frac{\partial V_z}{\partial y} - \mathbf{i}_y \frac{\partial V_z}{\partial x},$$

$$\nabla \phi = \mathbf{i}_x \frac{\partial \phi}{\partial x} - \mathbf{i}_y \frac{\partial \phi}{\partial y}. \tag{3}$$

In this case, solving an ill-posed system equations with a rectangular system matrix is essentially required because a number of the known field components is less than those of vector and scalar potentials. Discretization of Eq.(1) suggests that the vector $\mathbf{F}$ can be expressed by a sum of rotational $\mathbf{F}_V(=\nabla\times\mathbf{V})$ and a divergent $\mathbf{F}_S(=-\nabla\Phi)$ vector fields as

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_V + \mathbf{F}_S,\\ &= D_V\mathbf{V} + D_S\Phi,\\ &= \begin{pmatrix} D_V & D_S\end{pmatrix}\begin{bmatrix}\mathbf{V}\\ \Phi\end{bmatrix},\\ &= A\mathbf{f},\end{aligned} \tag{4}$$

where $A$ is a rectangular system matrix composed of the rotational $D_v$ and divergent $D_s$ operators. Namely, a number of unknowns is larger than those of equations.

### *Minimum Norm Solutions*

One of the ways to solve the ill-posed system of equations (4) is the minimum norm method. The minimum norm method yields a unique solution vector whose norm takes a minimum value. Minimum norm solution of Eq. (4) is given by

$$\mathbf{f} = A^T\left(AA^T\right)^{-1}\mathbf{F}. \tag{5}$$

Thus, the minimum norm solution (5) gives the vector $\mathbf{V}$ and scalar $\Phi$ potential vectors in (4). Taking a rotation of the vector potentials yields the rotational field components of the observed vector fields. Also, taking a gradient of the scalar potentials yields the divergent field components of the observed vector fields.

## SIMULATIONS

Fig. 2(a) shows an observed vector field. Figs. 2(b) and 2(c) show the vector and scalar potential distributions, respectively. Figs. 3(a) and 3(b) are the rotational and divergent field components, respectively. Fig. 3(c) is the recovered vector fields by the summation of rotational and divergent fields, which exactly corresponds to the observed one. By comparison of the results shown in Figs. 3(a) and 3(b), it has been confirmed that the rotational fields in Fig. 3(a) are the major vector components. Thereby, evaluation of the rotational field vectors makes it possible to reduce the unnatural components of the observed vector distribution.

Fig. 4(a) shows an observed vector field. Figs. 4(b) and 4(c) show the vector and scalar potential distributions, respectively. Figs. 5(a) and 5(b) are the rotational and divergent field components, respectively. Fig. 5(c) is the recovered vector fields by the summation of rotational and divergent fields, which exactly corresponds to the observed one. The result in Fig. 5(b) suggests that the divergent field components are the dominant vectors at both edges of the series currents where the current flows into or out of the observed field. On the other hand, the result in Fig. 5(a) suggests that the rotational field components are the dominant vectors at the central part of the series currents where the current is continuously flowing.

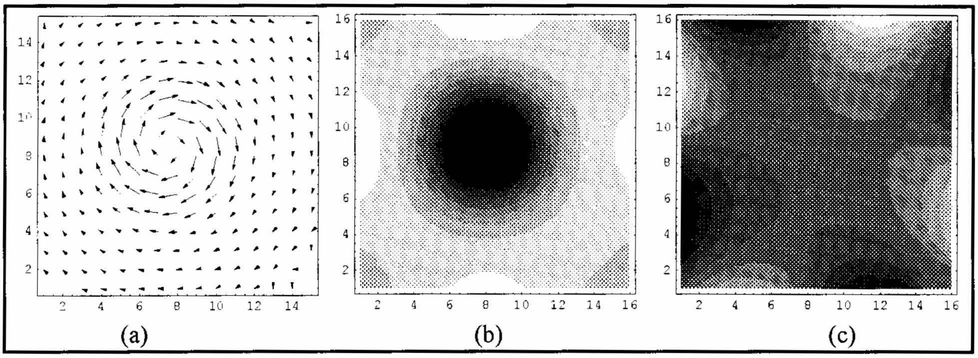

FIG.2. An example. (a)Observed vector, (b)vector potential, and (c)scalar potential distributions.

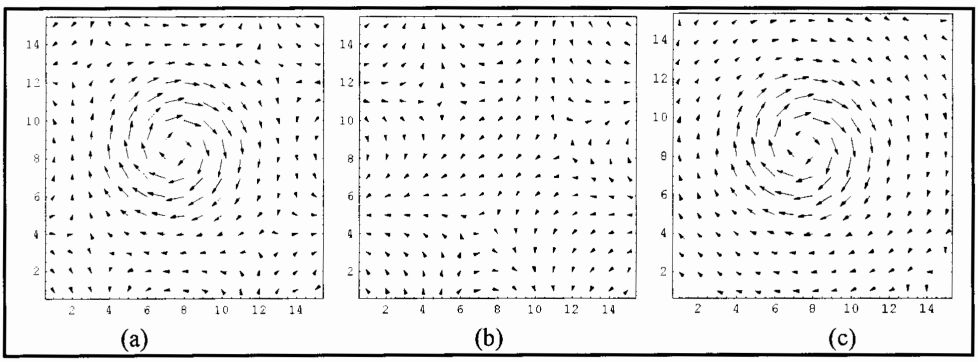

FIG. 3. (a) Rotational vector fields, (b) divergent vector fields, and (c) recovered vector fields obtained by the summation of rotational and divergent fields.

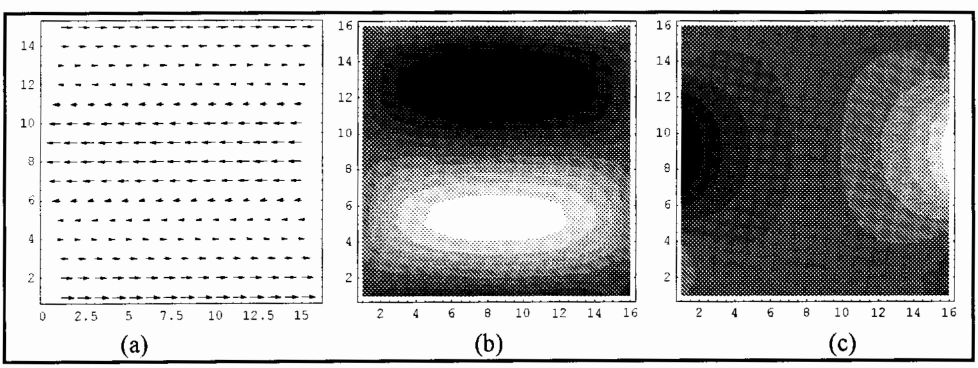

FIG.4. An example. (a)Observed vector, (b)vector potential, and (c)scalar potential distributions.

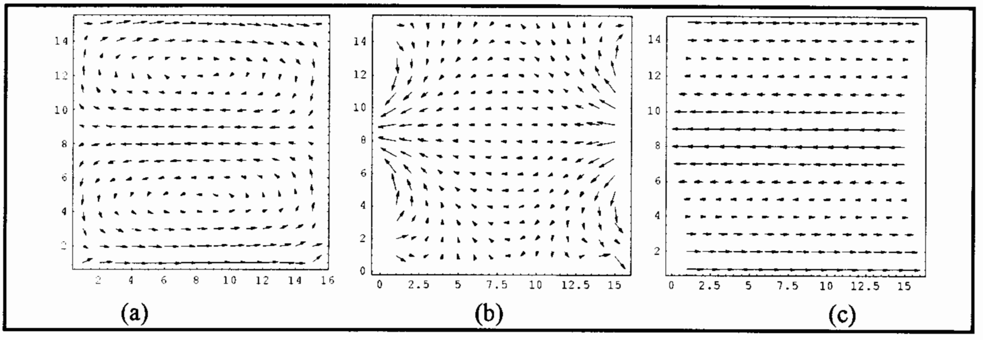


FIG. 5. (a) Rotational vector fields, (b) divergent vector fields, and (c) recovered vector fields obtained by the summation of rotational and divergent fields.

## APPILICATIONS

### *Eddy Current Searching*

Figs. 6(a) and 6(b) show the schematic diagram and measured magnetic field distribution caused by eddy current on a metal plane, respectively. By using a pick-up coil, the magnetic fields were measured at equi-spaced 100 (10×10) points on a 100×100mm$^2$ surface located in parallel to the target metal plate. A distance between the field measurement and metal plane surfaces was 5mm. The frequency of measured magnetic fields was 1MHz. Fig. 6(c) shows the eddy current distribution calculated from the magnetic field distribution in Fig.6(b).

Figs. 7(a) and 7(b) are the rotational and divergent field components of the eddy currents in Fig. 6(c), respectively.

The rotational field component in Fig. 7(a) corresponds to the eddy current component to be classified. As a result, the results in Fig. 7 demonstrated that unnatural components in the estimated eddy currents could be removed by our post processing.

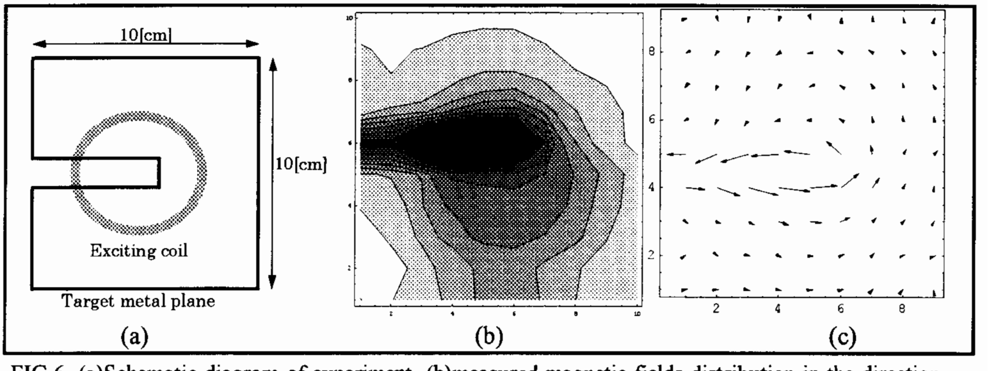


FIG.6. (a)Schematic diagram of experiment, (b)measured magnetic fields distribution in the direction of normal to the surface, and (c)eddy current distribution.

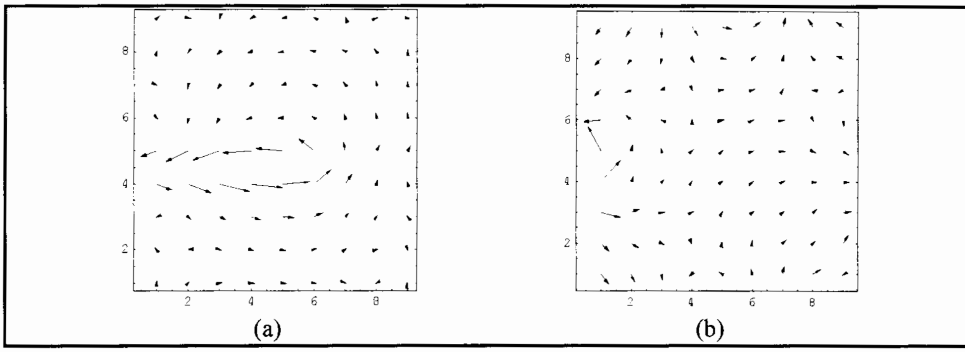


FIG. 7. (a) Rotational, and (b) divergent vector fields.

## *Magnetic Field Source Searching*

Figs. 8(a) and 8(b) show the schematic diagram and measured magnetic field distribution caused by magnetic field sources, respectively. Both of a loop antenna and a micro strip line were employed as magnetic field source. By using a pick up coil, the magnetic fields were measured at equi-spaced 441 (21×21) points on a 200×200mm$^2$ surface located in parallel to the target surface. A distance between the field measurement and target surfaces was 10mm. The frequency of measured magnetic fields was 20MHz. Fig. 8(c) shows the magnetic field source distribution estimated from the magnetic field distribution in Fig.8(b).

Figs. 9(a) and 9(b) are the rotational and divergent field components of the magnetic field sources in Fig. 8(c), respectively.

By considering the experimental results in Fig.9, classifying the rotational field components makes it possible to identify the major source component vectors. Also, the result in Fig. 9(b) suggests that the divergent field components are the dominant vectors at the edge of the target region where the current flows into or out of the observed field. Thus, in this magnetic field source searching problems, the divergent field components correspond to the unnatural components of magnetic field sources in Fig.8(c).

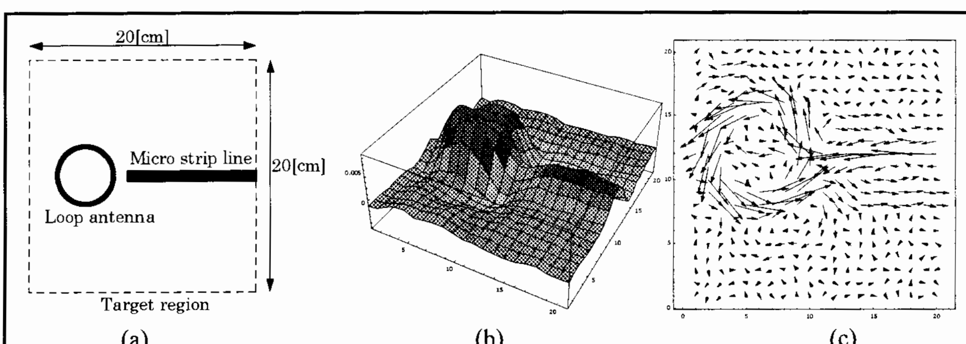


FIG.8. An experimental result of magnetic field source searching. (a)Schematic diagram of experiment, (b)measured magnetic fields in the direction of normal to the surface, and (c)estimated magnetic field source distribution.

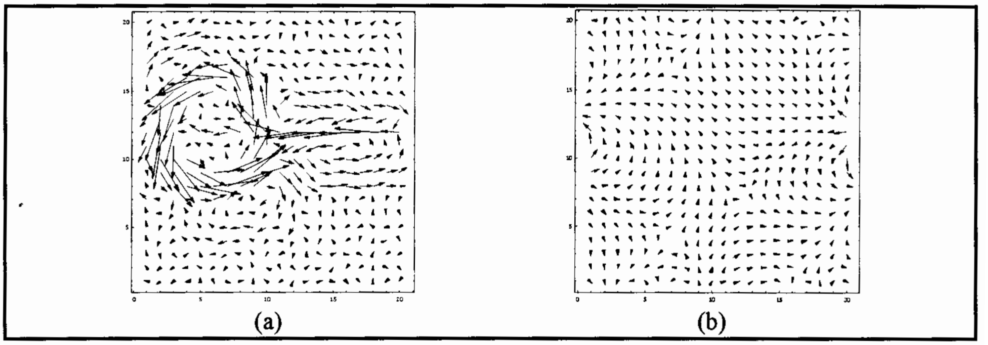

FIG.9. (a) Rotational vector, and (b) divergent vector fields.

## CONCLUSION

In the present paper, the new method for classifying scalar and vector potentials out of known vector fields have been proposed. The numerical simulations have verified the validity of the method.

As the concrete examples, our method has been applied to the magnetic field source searching problems, which are both experimental examinations of the eddy current and the magnetic field source classification.

As a result, it has been revealed that our method is capable of evaluating the vector and scalar potentials from the given vector fields. Thus, our method can be applicable to the post processing scheme for removing the unnatural noise vectors from the noisy solution vectors.

## REFERENCES

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