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Coil impedance computation having arbitrary geometrical shape

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Abstract

Coil impedance computation having arbitrary shape is relatively difficult task, because the inductance as well as resistance are not simple functions of the geometrical parameters but frequency.

In the present paper, we propose a new semi-analytical approach to compute the impedance vs. frequency characteristics of the coil having arbitrary shape. Any coils having complex geometrical shape can be divided into small conductors having simple geometrical shape. Applying analytical formula to each of the small subdivided conductors, each of the inductances and resistances is easily calculated by an analytical approach. Combining entire inductances and resistances taking into account the mutual inductances yields an equivalent circuit of the original coil having complex geometrical shape. Establishing and solving a set of circuit equations about this equivalent circuit make it possible to evaluate the impedance vs. frequency characteristics of the coil.

Simple examples verify our proposed methodology.

I. INTRODUCTION

To reduce the size and weight, operating frequency of the modern DC to DC converters is rising up a higher frequency[1,2]. Because of the skin and proximate effects, the coil inductance as well as resistance do not take a constant value decided by the medium parameter and geometrical shape. Namely, the coil impedance should be considered as the variables depending on the geometrical shape as well as operating frequency. Further, in order to realize the compact and light weight DC to DC converters, the coils have to be reformed to the relatively complex geometrical shapes. Thus, the coil impedance calculation in a classical approach confronts to a serious difficulty.

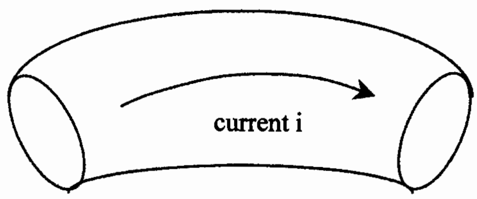
On the other side, development of the modern digital computers stimulates the various powerful numerical methods, e,g. finite element and integral equation methords[3,4]. These numerical methods work well to the distributed parameter systems such as the antenna and liner motor field computations, but it is required the enormous computational resource and skillfulness to compute the coil impedance having arbitrary geometrical shape.

In the present paper, we propose a semi-analytical coil impedance computation method. This method is based on a combination of the classical analytical method and modern discretizing approach. The classical approach yields always the exact result but it is only available to the coils having a relatively simple geometrical shape. On the other side, the modern numerical methods, in principle, can be applicable to the coils having any geometrical shapes. Application of the modern numerical methods to the coil impedance computation requires a great deals of knowledge for the partial differential equations and discretizing techniques, i.e.,mesh generation and setting the boundary conditions.

Our approach proposed here employs both classical and modern approaches. Any coil having complex geometrical shape can be subdivided into the small parts having a simple geometrical shape. Thereby, the classical analytical method can be applicable to each of the parts. Consideration of the mutual inductance among the parts leads to a establishment of a system of equations. Solving the system equations yields a coil impedance as the functions of both geometrical shape and operating frequency. Fairly good agreements among the computed, analytical and experimented results were obtained.

II. IMPEDANCE VS. FREQUENCY CHARACTERISTIC

A. Principle Let us consider a current carrying coil shown in Fig.1(a). This coil can be represented by a combination of the subdivided small conductors. Also, it is assumed that each of the subdivided conductors takes a similar geometrical shape shown in Fig.1(b). Since the current in the original conductor flows in parallel in a tangential direction of conductor, it is possible to assume an equivalent circuit as shown in Fig. 1(c).



(a)

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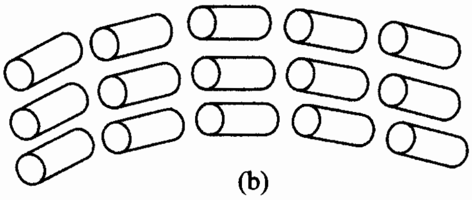

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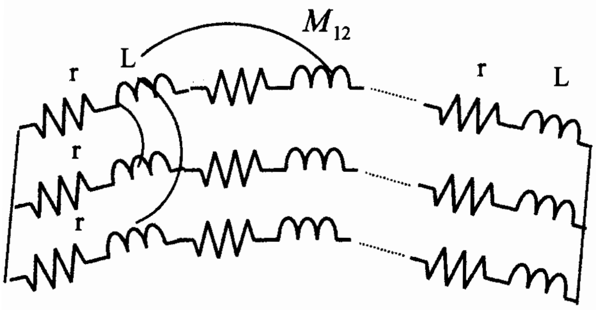


(c)

Fig.1 A current carrying coil and its modeling. (a)Original, (b)discretized model and (c)equivalent circuit.

Since the geometrical shape of a subdivided small conductor in Fig. 1(b) is a simple cylindrical form, it is possible to apply the classical formulae for the inductance and resistance calculations. Thus, the resistance r_i, self inductance L_i and mutual inductance M_{ij} (i=1,2,3,.., n, j=1,2,3,..,n, i$\neq$j) in Fig. 1(c) are respectively given by

$$r_i = \sigma \frac{s_i}{l_i}, \quad L_i = \frac{\mu_0}{2\pi}\left[\log\frac{2l_i}{a_i} - \frac{3}{4}\right],$$
$$M_{ij} = \frac{\mu_0}{4\pi}\int_{l_i}\int_{l_j}\frac{d\mathbf{l}_i \cdot d\mathbf{l}_j}{r_{ij}}, \quad i \neq j, \tag{1}$$

where σ, μ_0, s_i, l_i, a_i, r_{ij}, are the resistivity, permeability, cross-sectional area normal to the current path, length of current path, radius of an i-th conductor; mean distance between i and j-th conductors, respectively. Obviously, these equations have been derived by assuming a uniformly distributing current density in each of the subdivided conductors.[4,5)]

By means of the equivalent circuit shown in Fig. 1(c), it is possible to derive a following system of equations:

$$\mathbf{V} = Z\mathbf{I}, \tag{2}$$

where

$$\mathbf{V} = [v \quad v \quad \cdot \quad \cdot \quad v]^T,$$
$$\mathbf{I} = [i_1 \quad i_2 \quad \cdot \quad \cdot \quad i_n]^T,$$
$$Z = \begin{bmatrix} r_1 + j\omega L_1 & j\omega M_{12} & \cdot & \cdot & j\omega M_{1n} \\ j\omega M_{21} & r_2 + j\omega L_2 & \cdot & \cdot & j\omega M_{2n} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ j\omega M_{n1} & j\omega M_{n2} & \cdot & \cdot & r_n + j\omega L_n \end{bmatrix}, \tag{3}$$

In Eq. (3), $\mathbf{V}$, $\mathbf{I}$, v and i_i are the voltage vector, current vector with order n; impressed voltage and current in each of the small conductors, respectively.

Thus, the impedance z_{imp} of the conductor shown in Fig. 1(a) is given by

$$z_{imp} = \frac{v}{E_n^T \mathbf{I}} = \frac{v}{E_n^T Z^{-1}\mathbf{V}} = \frac{1}{E_n^T Z^{-1} E_n}, \tag{4}$$

where

$$\mathbf{E}_n = [1 \quad 1 \quad \cdot \quad \cdot \quad 1]^T \tag{5}$$

B. Experimental To verify our approach, we applied our method to the three types of coils. One is a simple cylindrical coil, the second is a thin film conductor and the third is a square loop coil composed of a thin film conductor.

The first coil shown in Fig. 2 is to compare the result of our approach with analytical one.

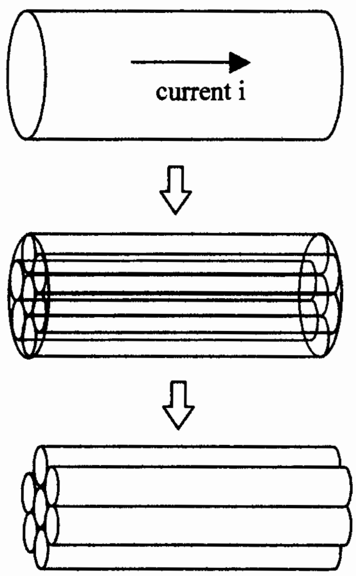


Fig. 2 . Modeling of a cylindrical coil.

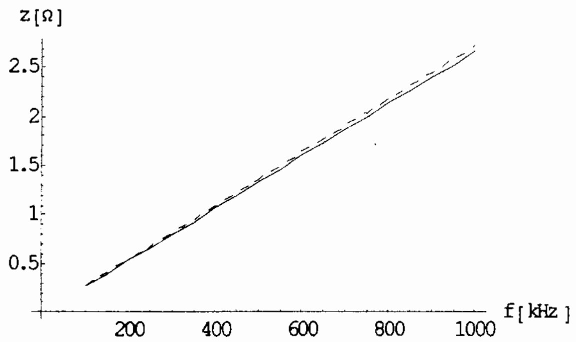
Table 1. Various constants used in the computation of a cylindrical coil.

Material	Copper
Length	60[cm]
Diameter	2.4[cm]
Radius of the subdivided conductor	0.1[cm]
Number of the subdivisions	145
Resistivity	$1.72\times10^{-8}[\Omega\cdot m]$

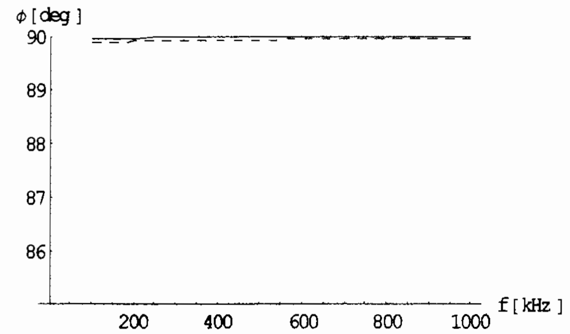
As shown in Fig. 2, the cylindrical coil is subdivided into a large number of cylindrical conductors. A cross-sectional area in each of the subdivided conductors is determined by dividing cross-sectional area of the original cylindrical coil by a number of subdivisions.

Table 1 lists the various constants used in the computations.

Figure 3(a) shows an impedance vs. frequency characteristic of this cylindrical coil together with the analytical one. Also, Figure 3(b) shows a phase vs. frequency characteristic together with analytical one.



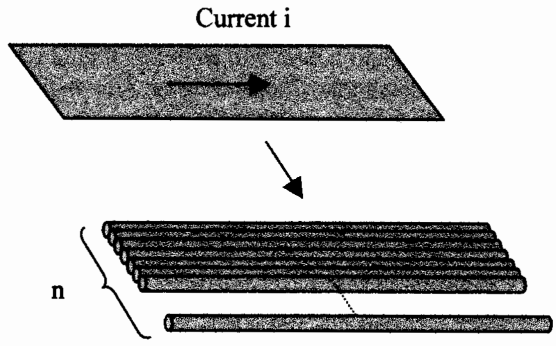
(a)



(b)

Fig. 3. Impedance vs. frequency characteristics of a cylindrical conductor.
(a) |Z| vs. frequency, and (b) phase angle vs. frequency.

By considering the results in Fig. 3, it is revealed that the computed values are well corresponding to the analytical ones.



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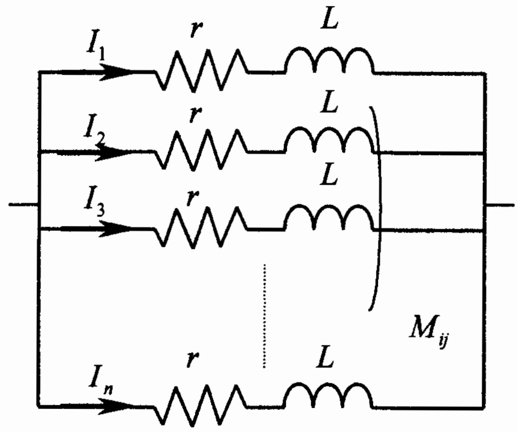


(b)

Fig. 4 . Modeling of a film coil.
(a) Discretization of film coil,
(b) Equivalent circuit.

Table 2. Various constants used in the computation of a film coil.

Material	Copper
Sizes	Length 60[cm] Width 10[cm]
Radius of subdivided conductor	4[mm]
Number of subdivisions	50
Resistivity	$1.72\times10^{-8}[\Omega\cdot m]$

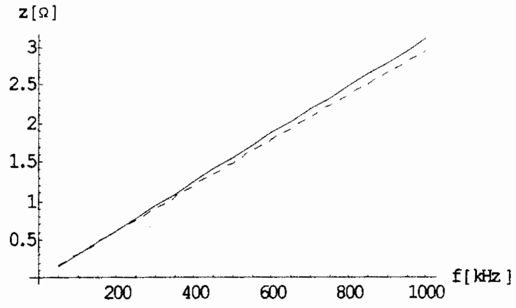
The second example was selected a film coil. This is because most of the power electronic devices are composed of the film conductors.

As shown in Fig. 4, the film coil is subdivided into a large number of cylindrical conductors. A cross-sectional area in each of the subdivided conductors is determined by dividing cross-sectional area of the original film coil by a number of subdivisions.

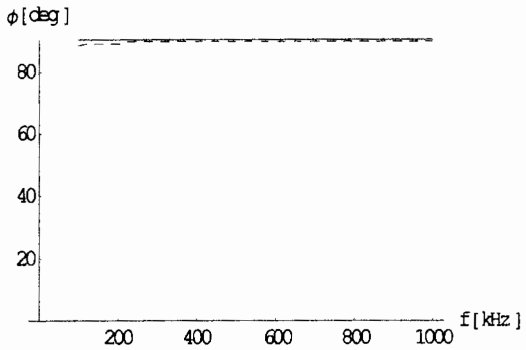
Table 2 lists the various constants used in the computations.

Fig. 5(a) shows an impedance vs. frequency characteristic of this cylindrical coil together with the experimental one. Also, Fig. 5(b) shows a phase vs.

frequency characteristic together with experimental one.



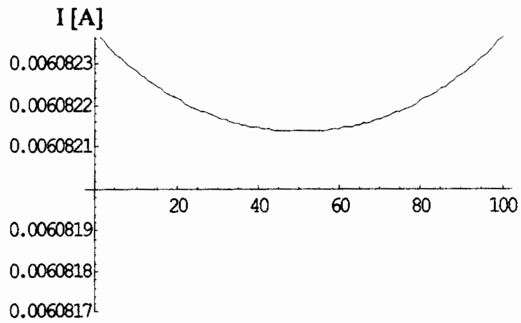
(a)



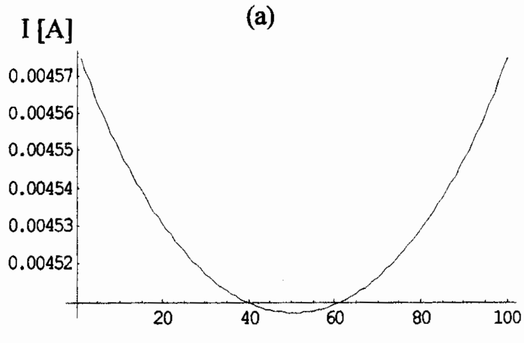
(b)

Fig. 5. Impedance vs. frequency characteristics of a film conductor.
(a) |Z| vs. frequency, and (b) phase angle vs. frequency.

By considering the results in Fig. 5, it is revealed that the computed values are well corresponding to the experimental ones.



(a)



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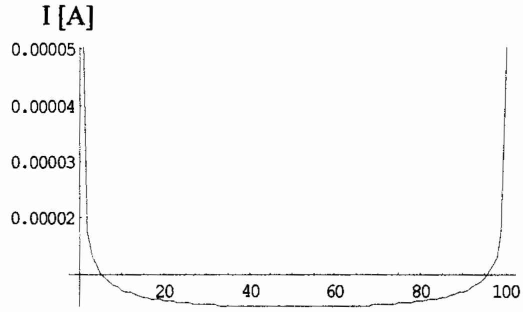

(c)

Fig. 6. Current distributions of a film conductor at different exciting frequencies.
(a) 50Hz, (b) 1kHz, (c) 1MHz.

In order to examine the skin effect in the film coil, we computed the current distributions at different frequencies. Figure 6 shows the current distributions in the film coil. According to the results in Fig. 6, localization of the current depends on the exciting frequency. This means that a wide film coil is a meaningless use for a high frequency excitation.

Original

Experimental & Simulation models
Fig. 7 . Modeling of a square loop coil composed of a thin film conductor.

Table 3. Various constants used in the computation of a square loop coil composed of a film conductor.

Material	Copper
Sizes	Length 10[cm], Sizes 10[cm], and height 1[cm]

Radius of the square loop coil composed of a film conductor	0.2[mm]
Number of the square loop coil composed of a film conductor	50
Resistivity	$1.72\times10^{-8}[\Omega\cdot m]$

Final example was a square loop coil composed of a thin film conductor. Most of the inductances are composed of the loop coils in order to increase a linkage magnetic flux.

Thus, it is extremely important to evaluate the impedance vs. frequency characteristics of the loop coils.

As shown in Fig. 7, the square loop coil is subdivided into a large number of square loop coils having cylindrical cross-sectional shape. A cross-sectional area in each of the subdivided conductors is determined by dividing cross-sectional area of the original film conductor by a number of subdivisions.

In this example, we worked out a practical simulation model which was composed of the enameled square loop coils with 50 turns. Table 3 lists the various constants used in the computations.

Fig. 8(a) shows an impedance vs. frequency characteristic of the square loop coil together with the experimentally simulated and experimental ones. Also, Fig. 8(b) shows a phase vs. frequency characteristic together with experimentally simulated and experimental ones.

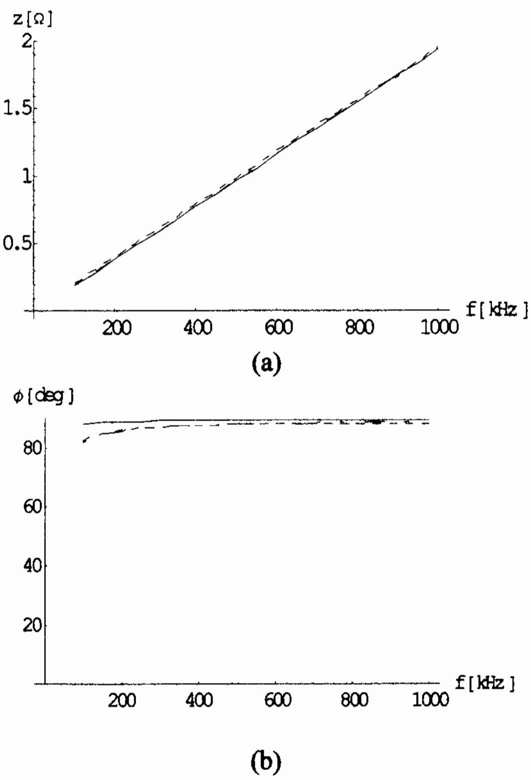

Fig. 8. Impedance vs. frequency characteristics of the square loop coil composed of a film conductor.

(a) |Z| vs. frequency, and (b) phase angle vs. frequency.
Solid, dotted and one point dotted lines refer to the computed, experimentally simulated and experimental results, respectively.

From the results in Fig. 8, it is obvious that the computed values are well corresponding to the experimentally simulated as well as experimental ones.

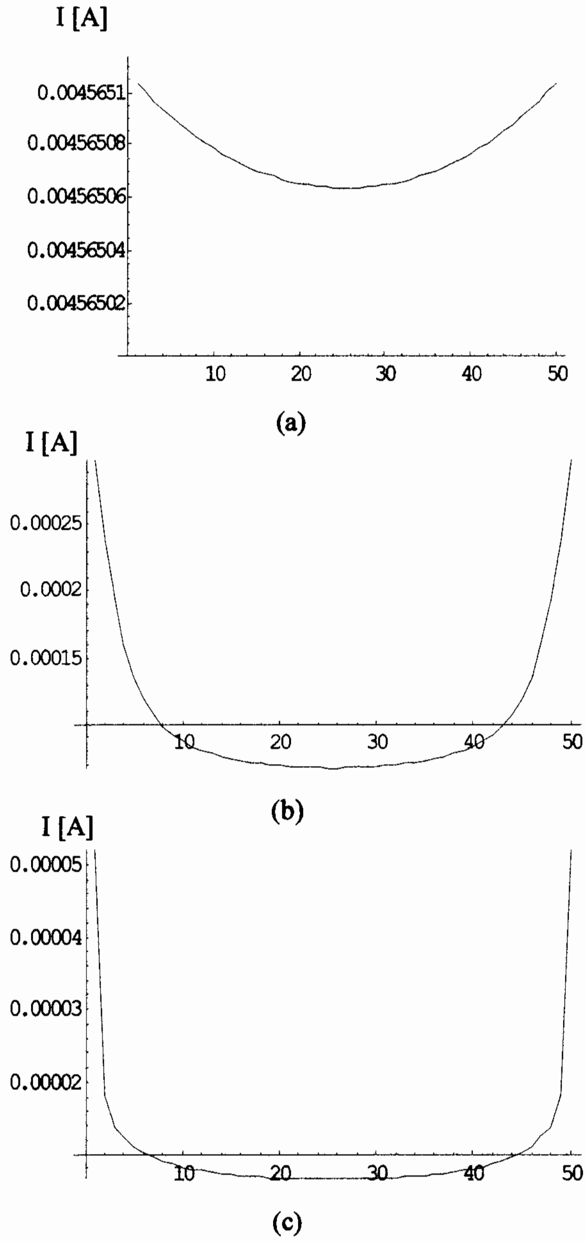

Fig. 9. Current distributions of the square loop coil composed of a film conductor at different exciting frequencies.
(a) 50Hz, (b) 1kHz, (c) 1MHz.

Figure 9 shows the current distributions at the cross section of film conductor. Comparison the current distributions in Figs. 6 with 9 suggests that the skin effect of the square loop coil is larger than those of the simple flat shape film coil. This difference is caused by an existence of a main flux penetrating the square loop.

III. CONCLUSION

As shown above, we have proposed the semi-analytical impedance computation method and succeeded in obtaining the fairly good results.

One of the merits of our methodology is that our approach can be easily extend to the entire DC to DC converter circuits. This may be possible to evaluate a stray leakage inductance.

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